2002 ME Graduate Student Conference
April 13, 2002

NODAL CONTROL OF A VIBRATING BEAM

Akshay N Singh
Ph.D. Candidate

Faculty Advisors: Dr. Y. M. Ram and Dr. Su-Seng Pang

ABSTRACT

Vibration control is an important engineering problem and many methods for both active and passive vibration absorption have been developed. This paper deals with developing a method to achieve nodal control at the point of excitation in a Bernoulli-Euler beam. Singh and Ram in [3] have shown that under certain conditions that have been characterized in [3] the steady state motion of a certain degree of freedom in a harmonically excited conservative system may be absorbed by both passive and active means. Ram in [1] has developed a method to eliminate the steady state motion of a prescribed location in a continuous system like rod under the influence of a harmonic excitation. He has presented a closed form solution for the control gain in terms of infinite product of eigenvalues. This thesis extends the approach in [1] to achieve nodal control for suppressing vibration at prescribed location in beams and provides a simpler formula for the control gain in terms of eigenfunctions. It is established that, for a uniform Bernoulli-Euler beam, the steady state motion at the point of excitation can be absorbed by means of a control force determined from displacement information at the point of application. A closed form solution for the control gain is presented and a criterion for implementing the control by active and passive means is developed. The result for the control gain is generalized for the case of a non-uniform beam. It is also shown through some examples that the theory can be also applied to eliminate the steady state motion at any desired location other than the point of excitation. Analysis is also performed to determine the optimal control force and investigate the stability of the overall system. Several controllability graphs are shown and meaningful conclusions are drawn from these graphs. An experiment is designed to validate the proposed theory and display its practicality.

The developed theory will provide a strong foundation for realizing realistic and convenient methodologies in control applications in cases like surgical procedures, drilling and turning operations etc. However, one of the many direct applications of this method is structural vibration control in an aircraft wing. Several measurements such as vibrational response, air temperature, wind velocity etc are required in order to monitor flight conditions in an aircraft. These data also assist the pilot in flying the aircraft. Sensors and data collection circuitry form an entire network of the electrical wiring all in and around the airplane body. Data acquisition devices are also located on the wing of the airplane. Shielding of these devices from undesirable vibration of the wing is critical in order to avoid noise in the gathered data and prevent physical damage. Exclusion of steady state vibration at the locations of these devices provides the motivation for this investigation.

Consider a uniform Bernoulli-Euler beam of length L. Suppose that the beam is excited by a harmonic force $f(t)=\cos\omega t$, as shown in Figure 1(a). The steady state motion of a prescribed point of the beam may be vanished by applying a concentrated control force $u(a,t)$ at $x=a$ as shown in Figure 1(b).

$$u(a,t)=\alpha w(a,t), \qquad (1)$$

where α is the control gain. The partial differential equation for the controlled Bernoulli-Euler beam shown in Figure 1(b), for $0<x<L$, $t>0$, is given as

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 w}{\partial x^2}\right)+A\rho\frac{\partial^2 w}{\partial t^2}=\delta(x-a)\alpha w, \qquad (2)$$

and the boundary conditions are

$$w(0,t)=0 \qquad \text{no deflection,} \qquad (3)$$

$$\frac{\partial w(0,t)}{\partial x}=0, \qquad \text{no slope,} \qquad (4)$$

$$\frac{\partial^2 w(L,t)}{\partial x^2} = 0, \qquad \text{no moment,} \qquad (5)$$

$$\frac{\partial}{\partial x}\left(EI\,\frac{\partial^2 w(L,t)}{\partial x^2}\right) = \cos wt \qquad \text{shear force.} \qquad (6)$$

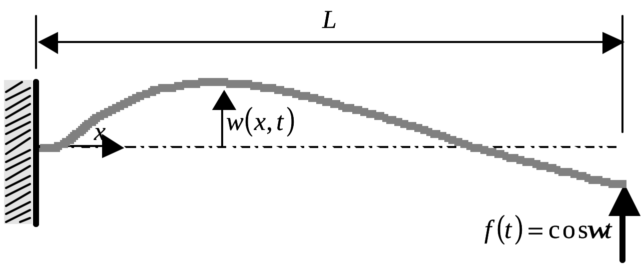
The work here focuses on determining a closed form solution for the control gain a that absorbs the motion of the beam at $x = L$.

A closed form expression for the control gain a obtained from mathematical manipulation is expressed as

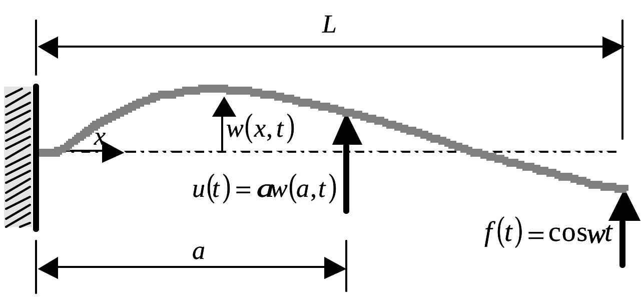
$$a(a,w) = EI\left(\frac{v_2'''(a) - v_1'''(a)}{v_1(a)}\right), \qquad (7)$$

where v_1 and v_2 are the deflections of the beams with span $0 < x < a$ and $a < x < L$ respectively. The values of the control gain $a(a,h)$ for $a = 0.2$, as function of the non-dimensional parameter $h = bL$ are shown in Figure 2.

FIGURES



(a) Uncontrolled beam



(b) Controlled beam

Figure 1: Vibration control of a harmonically excited beam

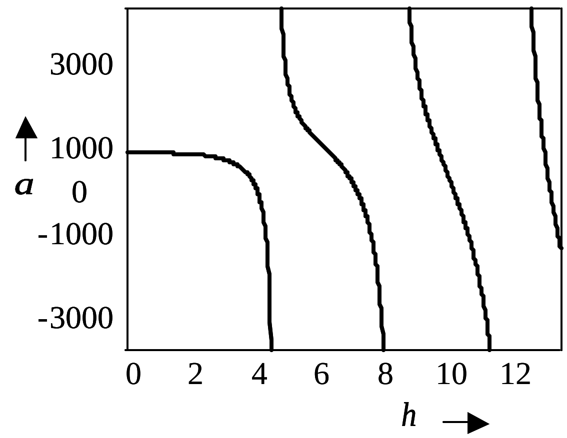


Figure 2: Plot of control gain a against non-dimensional parameter h

ACKNOWLEDGMENTS

The research was supported in part by a National Science Foundation research grant CMS-9978786.

REFERENCES

1. Ram Y. M., Nodal Control of a Vibrating Rod, Mechanical Systems and Signal Processing, 2002 (to appear).
2. Den Hartog J. P., Mechanical Vibrations, 4th edition, McGraw Hill, 1956.
3. Singh K. V., Ram Y. M., Dynamic Absorption by Passive and Active Control, Journal of Vibration and Acoustics, Vol. 122 (4), pp. 429-433, October 2000.
4. Inman D. J., Engineering Vibration, Prentice Hall, 1994.
5. Ram Y. M., Elhay S., The Theory of a Multi Degree of Freedom Dynamic Absorber, Journal of Sound and Vibration, pp. 607-615, Vol. 195, 1996.
6. Singh K. V., Ram Y. M., Numerical Algorithm For The Transcendental Eigenvalue Problem, AIAA Journal (accepted).
7. Elhay S., Gladwell G.M.L. Golub, G.H. and Ram Y.M., On some Eigenvalue-Eigenvector Relations, SIAM Journal on Matrix Analysis and Applications, 20(3), pp. 563-574, 1999.
8. Mottershead J.E., On the zeros of structural frequency response functions and their sensitivities, Mechanical Systems and Signal Processing, Vol. 12, pp. 591-597, 1998.
9. Mottershead J.E. and Lallement G., Vibration nodes, and the cancellations of poles and zeros by unit-rank modifications to structures, Journal of Sound and Vibration, Vol. 222, pp.833-851, 1999.