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PHYSICAL PARAMETER ESTIMATION OF VIBRATING STRUCTURE FROM ITS SPECTRAL DATA: A NEW MATHEMATICAL MODEL

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ABSTRACT

The problem of reconstructing a model with prescribed spectral data is known as inverse eigenvalue problem. Reconstruction of the distribution of physical parameters of a continuous vibratory system by using its eigenvalues is addressed here. Considering a unit length piecewise continuous rod as shown in figure 1. The eigenvalue problem associated with this rod is given by the following set of differential equations

$$\begin{cases} pu'' + lqu = 0, \text{ where, } l = w^2 \\ rv'' + lsv = 0, \text{ where, } l = w^2 \\ u(0) = 0 \qquad v'(L) = 0 \\ u(a) = v(a) \qquad u'(a) = gv'(a) \end{cases} . \tag{1}$$

Applying the boundary and matching conditions of displacement and force leads to problem of finding the non-trivial solution of

$$\begin{bmatrix} \sin\frac{wa}{a} & -\sin\frac{wa}{b} & -\cos\frac{wa}{b} \\ \frac{w}{a}\cos\frac{wa}{a} & -\frac{gw}{b}\cos\frac{wa}{b} & \frac{gw}{b}\sin\frac{wa}{b} \\ 0 & \frac{w}{b}\cos\frac{wL}{b} & -\frac{w}{b}\sin\frac{wL}{b} \end{bmatrix} \begin{pmatrix} z_2 \\ z_3 \\ z_4 \end{pmatrix} = \mathbf{o} . \tag{2}$$

We named this problem the *Transcendental Eigenvalue Problem* (TEP). The general form of this problem is

$$\mathbf{A}(w)\mathbf{z} = \mathbf{o} . \tag{3}$$

Frequently the classical finite element and finite difference formulation are used in approximating such a continuous system. The characteristic equation of the obtained eigenvalue problem is a polynomial. In contrast, the continuous systems are characterized by TEP [1]. By using finite element or finite difference method, the TEP is transformed into an algebraic eigenvalue problem. It has been concluded by [2,3] that the solution to the discrete problem is not a good approximation to the continuous one. Past research associated with inverse problems of the continuous vibratory system can be found in [4,5,6,7,8]. Since the behavior of a finite dimensional polynomial is fundamentally different from the transcendental function, such an approach may involve inaccurate approximation of the physical parameters, as illustrated in figure 2.

For the given continuous system in figure 1, the inverse problem can be defined as follows:

Given *the resonant frequencies* w_1, w_2, *anti-resonant frequency* m_1 *and the total mass of the rod.*
Determine *the physical parameters* p_1, p_2 *and* q_1, q_2.

The problem now is of determining the roots of the system of transcendental frequency equations,

$$\begin{cases} F_1(a, b, g) = \det\left(\mathbf{A}(a, b, g)\big|_{w=w_1}\right) = 0 \\ F_2(a, b, g) = \det\left(\mathbf{A}(a, b, g)\big|_{w=w_2}\right) = 0 , \\ F_3(a, b, g) = \det\left(\mathbf{A}(a, b, g)\big|_{w=m_1}\right) = 0 \end{cases} \tag{4}$$

for the given values of w_1, w_2 and m_1. The research aims at developing low dimensional analytical models allowing estimation of the physical parameters of the structures from measured vibration test data. The main idea presented here is to replace the continuous system with variable physical parameters by a continuous system with piecewise uniform properties as shown in figure 3. The boundary and matching conditions between the various parts of the continuous model can be expressed in the TEP form. A rapidly converged algorithm is used for evaluation of the physical parameters of the system. The algorithm implements the Newton's iterative method for determining the physical

parameters of the system. Formulation of such mathematical models for non-uniform axially vibrating rods and reconstruction of their area distribution by using this algorithm, as illustrated in figure 4, is presented. This proposed solution of TEP can also be used to solve classical direct problems in structural dynamics such as buckling [9] and vibration control [10].

FIGURES AND TABLES

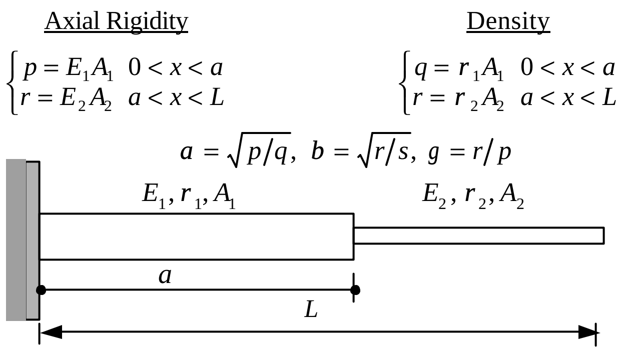

Fig.1. Piecewise continuous axially vibrating rod

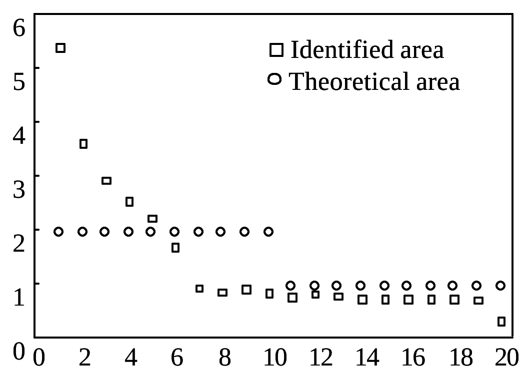

Fig.2.Physical parameter Identification of piecewise rod from its associated discrete model

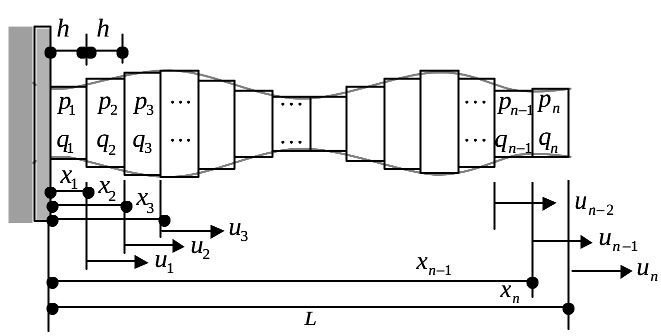

Fig.3. New mathematical model used for the approximation of a non-uniform rod

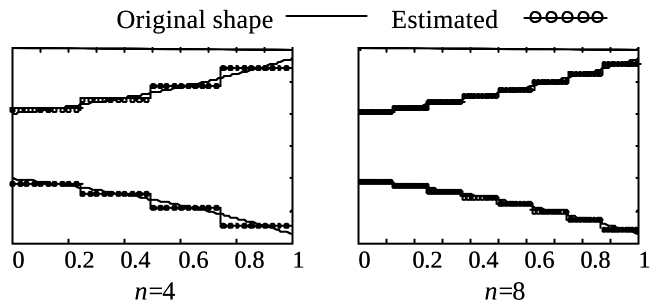

Fig.4. Reconstruction of the shape of the exponential rod with model order *n*=4 and *n*=8

ACKNOWLEDGMENTS

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