A DISJOINING PRESSURE FOR SMALL CONTACT ANGLES AND ITS APPLICATIONS

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ABSTRACT
A thin liquid film experiences additional intermolecular forces when the film thickness \( h \) is less than roughly 100 nm. The effect of these intermolecular forces at the continuum level is captured by the disjoining pressure \( \Pi \). Since \( \Pi \) dominates at small film thicknesses, it determines the stability and wettability of thin films. To leading order, a thin film can be treated as uniform and \( \Pi = \Pi(h) \). This form, however, cannot be applied to films with non-zero contact angles. A recent ad-hoc derivation to include the slope \( h_x \) leads to a \( \Pi = \Pi(h, h_x) \) that allows a contact line to move without slip \([1, 2]\). This work derives a new disjoining-pressure formula by minimizing the total energy of a drop on a solid substrate (Fig. 1). The minimization yields an equilibrium equation that relates \( \Pi \) to an excess interaction potential (Fig. 2), \( E = E(h, h_x) \). By considering a fluid wedge on a solid substrate, \( E(h, h_x) \) is found by pairwise summation of van der Walls potentials. This gives in the small-slope limit
\[
\Pi = \frac{B}{h^3} \left( \alpha^4 - h_x^2 + 2h_x^2h_{xx} \right),
\]
where \( \alpha \) is the contact angle and \( B \) is a material constant. The term containing the curvature \( h_{xx} \) is new; it prevents a contact line from moving without slip. For a uniform film, \( h_x = h_{xx} = 0 \), and \( \Pi = \pm B\alpha^4 / h^3 \), which recovers the expression used previously \([3, 4]\). Equilibrium drop and meniscus profiles are calculated for different \( B \), as shown in Figs. 3–6, where \( \varepsilon \) is the ratio of disjoining to capillary pressure. Evolution of a film step is solved by a finite-difference method with the new disjoining pressure included; it is found that \( h_{xx} = 0 \) at the contact line is sufficient to specify the contact angle (Fig. 7).
Figure 3. Equilibrium profiles of C=0 with symmetric central condition.

Figure 4. Equilibrium drop profiles.

Figure 5. Uniform-film grows to a meniscus.

Figure 6. Contact film grows to a meniscus.

Figure 7. Evolution profiles of the step thin film with fixed contact line at different time.

REFERENCES