



ASYMPTOTIC ADAPTIVE REGULATION OF PARAMETRIC STRICT-FEEDBACK SYSTEMS WITH ADDITIVE DISTURBANCE

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ABSTRACT

The class of systems to which adaptive control can be applied was vastly broadened with the advent of the integrator backstepping design [8]. This systematic design procedure allows one to adaptively stabilize systems that are in the so-called parametric strict-feedback form [8]. In the presence of bounded external disturbances, it is well known that the performance of adaptive controllers can significantly deteriorate and even lead to instability. In this case, the parameter estimate cannot be proven bounded, leading to the unboundedness of other closed-loop signals. Common approaches for counteracting this problem include adding a robustifying (leakage) term to the adaptation law (e.g., the σ -modification [5] and the ϵ 1-modification [10]), or using a projection operator [5,8] to confine the parameter estimate to a bounded convex set in the parameter space. Leakage modifications have the major disadvantage of not recovering the disturbance-free stability performance of the unmodified adaptation law if the disturbance disappears after some time. On the other hand, projection operators preserve the ideal properties of the adaptive controller if the disturbance disappears, but require parameter bounds to be known a priori. Generally, projection-based adaptation laws are discontinuous, which violates the Lipschitz condition for existence of classical solutions to differential equations. Furthermore, the discontinuity is not desirable from an implementation standpoint. This shortcoming however was addressed in [11] by introducing a boundary layer around the convex set that resulted in a Lipschitz continuous projection operator. Recently in [2], we proposed a smoothed version of the projection operator of [11], which replaces the Lipschitz continuity with the stronger property of n times continuous differentiability C^n while introducing minor or no modifications to the other projection properties. This new projection operator is useful for backstepping-based, robust adaptive controllers that require multiple differentiations of the adaptation law.

Robust adaptive control laws for systems affected by external disturbances can generally be shown to ensure the

boundedness of closed-loop signals, but not asymptotic tracking and regulation to zero. In [4], an adaptive backstepping controller with tuning functions for linear systems with output and multiplicative disturbances was redesigned with a switching σ -modification. The controller gives a tracking error proportional to the size of the perturbations. The tracking control problem for SISO nonlinear systems with unknown control coefficients and time-varying disturbances was recently studied in [3]. The robust adaptive controller proposed in [3] was shown to guarantee the global uniform boundedness of the tracking error.

In this paper, we consider a class of n th-order, multi-input/multi-output (MIMO), nonlinear parametric strict-feedback systems with matched, unknown, time-varying, additive disturbances and unmatched, uncertain, constant parameters with the form:

$$\begin{aligned}\dot{x}_1 &= \varphi_1^T(x_1)\theta + x_2 \\ \dot{x}_2 &= \varphi_2^T(x_1, x_2)\theta + x_3 \\ &\vdots \\ \dot{x}_n &= \varphi_n^T(x_1, \dots, x_n)\theta + d + u\end{aligned}\tag{1}$$

Our goal is to design a continuous adaptive controller that is insensitive to exogenous disturbances, and asymptotically drives the output to zero while maintaining all closed-loop signals bounded. Under the assumption that the disturbance is C^2 with bounded derivatives, we propose a modified adaptive backstepping design that exploits the new control mechanism of [11]. The control mechanism of [11] is reminiscent of the second-order SMC technique and the new C^n projection-based adaptation law of [2]. A Lyapunov-type stability analysis is used to prove semi-global asymptotic output regulation.

Simulation on a 4th order SISO system illustrates the main result:

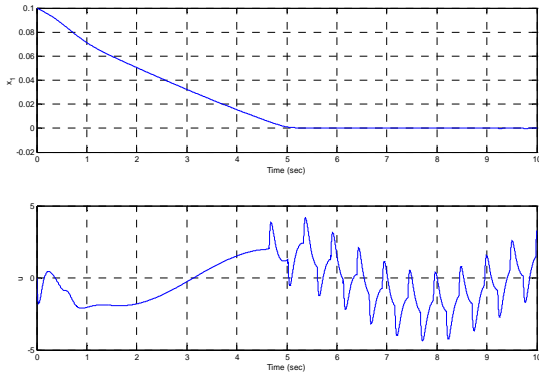


Figure 1: System output and control input

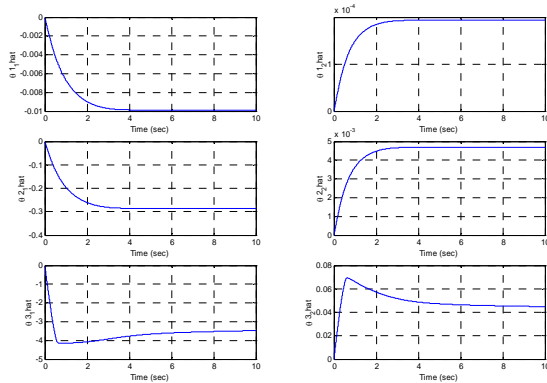


Figure 2: Parameter estimates

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