NATURAL FREQUENCY SENSITIVITY IN CONTINUOUS AND LAMINATED BEAMS

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ABSTRACT

Any moving parts will have vibration of some sort. Vibration becomes a problem when resonance occurs. Resonance can allow vibration to cause large deflections with minimal energy input. If left unchecked, resonance can cause failure in the structural members of a system. One solution is to design systems which minimize resonance. To do this, a designer must know the natural frequencies of the design, as it is at these frequencies where resonance will occur.

This work explores how changing various properties of a structural member changes the natural frequency of the design. Three methods of analysis are proposed to determine natural frequency sensitivity. Derivation to find a closed form solution was attempted first. This will be followed by finite element analysis and experimental verification.

Two models were used. For a general analysis, a simple continuous, isotropic cantilever beam of constant rectangular cross-section was used. Because composites are becoming so prevalent in engineering design work, the second model was chosen as a continuous laminated cantilever beam. Each layer of the beam is isotropic to simplify the calculations, but the different layers have different properties. These models are shown in Figure 1.

In theoretical calculations of a simple continuous rectangular beam, the Transcendental Eigenvalue Solution [1] can be used to solve the governing equation of Euler-Bernoulli beam vibration. Natural frequency in a beam of rectangular cross-section is sensitive to changes in elastic modulus ($E$), material density ($\rho$), and beam height ($h$):

$$\omega = \frac{1}{2\sqrt{3}} \beta^2 h \left( \frac{E}{\rho} \right)^{1/2},$$

where $\beta$ depends on the Transcendental Eigenvalue solution and is constant for a given length and boundary conditions. This gives the solution of vibration sensitivity due to the various parameters:

$$\frac{\partial \omega}{\partial E} = \frac{1}{4\sqrt{3}} \beta^2 h \left( \frac{1}{\rho E} \right)^{1/2},$$

$$\frac{\partial \omega}{\partial \rho} = -\frac{1}{4\sqrt{3}} \beta^2 h \left( \frac{E}{\rho^2} \right)^{1/2},$$

$$\frac{\partial \omega}{\partial h} = \frac{1}{2\sqrt{3}} \beta^2 \left( \frac{E}{\rho} \right)^{1/2}.$$
the summer. Two-dimensional finite element models will be used to compare an array of beams with differing elastic modulus, density, and height. The resulting first modes of natural frequency will be compared by graph and the slopes of the graphs analyzed for comparison with theoretical findings.

Beams will then be fabricated to match the conditions of the theoretical models and the frequency of vibration will be measured after each test subject is struck with a hammer. Since the impact of the hammer will be an impulse instead of a time-dependent forcing function, the resultant vibration will be at the natural frequency of the test subject. The results will be compared for tests subjects of differing parameters to compare with the theoretical and finite element models. These tests will verify whether two-dimensional models are sufficient for real world applications.

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REFERENCES