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ON THE THERMODYNAMIC ENTROPY OF DAMAGE EVOLUTION

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ABSTRACT

Permanent degradations are the manifestation of irreversible processes that disorder a system and generate entropy in accordance to the second law of thermodynamics. Disorder in systems that undergo degradation continues to increase until a critical stage when failure occurs. Simultaneous with the rise in disorder, entropy monotonically increases. Thus, entropy and thermodynamic energies offer a natural measure of component degradation [1, 2]. Fatigue damage is an example of a degradation process whereby the irreversible progression of cyclic plastic strain energy accumulates such that the system's entropy reaches its critical value at the onset of fracture. In the present study, thermodynamic entropy is utilized to show that the cyclic energy dissipation in the form of thermodynamic entropy can be effectively utilized to determine damage evolution during fatigue process.

EXPERIEMNTAL PROCEDURE

Fatigue tests are conducted with Aluminum 6061-T6 and Stainless Steel 304 specimens. Bending fatigue tests involve a plane specimen clamped at one end, and oscillated at the other end which is connected to the crank with specified amplitude and frequency.

High-speed, high-resolution infrared thermography is used to record the temperature evolution of the specimen during the entire experiment.

Fig. 1 shows a typical evolution of temperature plotted as a function of number of cycles from the beginning of the test until the specimen breaks. With the stress above the fatigue limit, the temperature evolution undergoes three phases: an initial increase (Phase 1) due to plastic deformation and hardening phenomenon, steady-state (Phase 2) due to thermal equilibrium with the environment, and an abrupt increase prior to failure (Phase 3) due to plastic deformation at the crack tips.

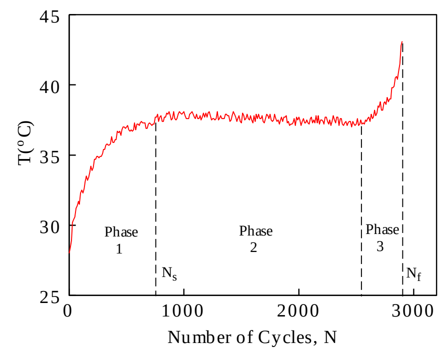

Fig. 1 Typical temperature evolution for a low-cycle fatigue test of Aluminum, $\delta = 49.53\,mm$.

THEORY AND FORMULATION

The variation of entropy is the summation of entropy generation inside the system ($\dot{s}_i$) and entropy flow exchange between the system and the surroundings ($\dot{s}_e$) [3].

$$\dot{s} = \dot{s}_i + \dot{s}_e \tag{1}$$

The Clausius-Duhem inequality states that in solids with internal friction all the deformations cause positive entropy production rate [4].

$$\dot{s}_i = \frac{1}{T}\sigma : \dot{\varepsilon}_p - \frac{1}{T}A_k \dot{V}_k - \frac{1}{T^2}\vec{q}.\overrightarrow{\nabla T} \geq 0 \tag{2}$$

The entropy production (Eq. 2) is a combination of three terms (entropy generation due to plastic deformation, internal variables evolution, and heat conduction respectively) that are often difficult to measure experimentally. Entropy flow, on the other hand, can be simply related to the experimentally measurable quantities such as temperature. The use of entropy flow instead of entropy production has been recently employed by other researches to quantify the degradation of components experiencing other type of degradation process ([1], [2]).

One can evaluate the rate of entropy flow to the surrounding by simple temperature variation:

$$\dot{s}_e = q/T = h(1 - T_a/T) \tag{3}$$

So, the volumetric entropy flow to the surrounding is:

$$s_e = \int_0^{t_f} h(1 - T_a/T)dt \qquad (4)$$

Fig. 2 shows the normalized entropy flow during the fatigue life until failure. It can be seen the relation between the normalized cycles to failure and normalized entropy is approximately linear and can be described as follows (a similar trend between normalized wear plotted against the normalized entropy was reported by [2]):

$$\frac{s_e}{s_f} \cong \frac{N}{N_f} \qquad (5)$$

Duyi and Zhenlin ([5]) developed an approach for analyzing the evolution of the low-cycle fatigue damage, particularly for estimation of the remaining fatigue lifetime. They used the reduction of static toughness and the plastic strain energy during fatigue failure to establish a damage variable defined as:

$$D_N = \frac{D_{(N_f-1)}}{\ln(N_f)} \ln(1 - N/N_f) \qquad (6)$$

According to Eq. (6), damage variable is a function of N/N_f the ratio, i.e.,

$$D = f(N/N_f) \qquad (7)$$

The damage variable in Eq. (7) can be directly related to entropy using Eq. (5) as follows.

$$D = f(s_e/s_f) \qquad (8)$$

Since the damage parameter is a measure of degradation in a material and that degradation itself is an indicator of the irreversibility in a system, a damage criterion must be a convex function to satisfy second law of thermodynamics. Also, two important properties of a convex function are that first and second derivative must be increasing.

The above discussion articulates that an equation for damage variable must satisfy the above convex function properties. Accordingly, the following expression for the damage variable expressed in terms of the entropy flow satisfies both of the convex function properties.

$$D_N = \frac{-1}{\ln(s_f)} \ln(1 - s/s_f) \qquad (9)$$

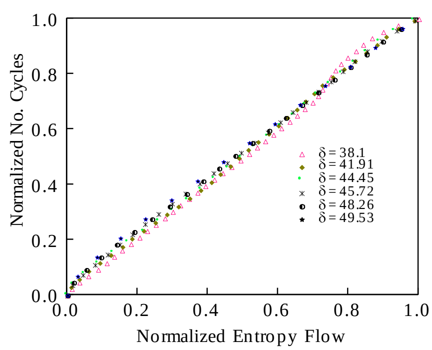

Fig. 2 Normalized number of cycle vs. normalized entropy

RESULTS AND DISCUSSION

Fig. 3 shows the variation of damage variable, *D*, at different displacement amplitudes along with the Eq. (9) for Al-6061. As shown in this figure, in the very early stage of fatigue life, *D* increases gradually. Then, the slope increases with increasing number of cycles followed by a rapid rise near the critical number of cycles that correspond to the fatigue life. It can be seen that the results of present work are in good agreement with the results of [5].

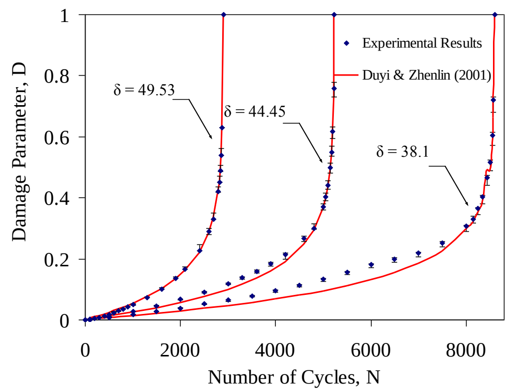

Fig. 3 Damage variable from present experimental work and the work of Duyi and Zhenlin for Al-6061-T6

CONCLUSION

The results of a series of bending fatigue experiments are presented that establish a simple and effective technique to evaluate damage evolution. It is shown that the entropy flow can be utilized to measure the degradation of the material under the cyclic bending loads. The experimental results provide a single correlation which gives the damage parameter as a function of entropy flow.

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