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MODEL-BASED CONTROL OF A NEW ACTIVE TILTING-PAD BEARING

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ABSTRACT

This work introduces a new type of active fluid-film bearing and its feedback control. In particular, we propose to actively adjust the angular velocity of the pads of a tilting-pad bearing in response to changes in the operating conditions of the rotating machine. This is motivated by the observation that there is more control authority in the pad tilt motion than in its radial translation. To this end, we first develop a dynamic model for the bearing system, inclusive of the nonlinear hydrodynamic force for the infinitely-short bearing case. A model-based controller is then constructed, based on measurements of the journal position and velocity and pad tilt angles, to ensure that the journal is asymptotically regulated to the bearing center. Numerical simulations illustrate the performance of the active bearing under the proposed control in comparison with the bearing's standard passive mode of operation.

Consider the tilting-pad bearing system shown in Figure1. The equations of motion for the journal are

$$m\ddot{q} = F(q,\dot{q},\alpha,\dot{\alpha},q_p,\dot{q}_p) + W \qquad (1)$$

where m is the constant journal mass, $q=(x,y)$ is the position of the journal center, $\alpha \in \Re^4$ is the vector of pad title angles, $q_p \in \Re^4$ represents the vector of pad radial positions, $F \in \Re^2$ denotes the total hydrodynamic force the fluid film applies on the journal, and $W \in \Re^2$ is the constant external load. Further, let $c = R - r$ be the nominal clearance (i.e., nominal fluid film thickness), L be the bearing axial length, and

$$S_i = \left[(i-1)\pi/2 - \theta_0, (i-1)\pi/2 + \theta_0\right], \quad i = 1,2,3,4 \qquad (2)$$

be the arc of the ith pad in terms of the θ coordinate where $\theta_0 > 0$.

To transform the tilting-pad bearing into an *active* system, control inputs for the system dynamics need to be selected. Here, we propose to utilize $\dot{\alpha}$ as the control input to (1), as opposed to the more common choice of $\dot{q}_p$ [1, 2]. This new active approach is primarily based on the following two observations: a) the wedge action has more pressure-generating capacity (i.e., control authority) than the squeeze action, and b) the pad tilting (resp., radial) motion has a direct effect on the wedge (resp., squeeze) action.

Now, the key issue in (1) is the model for the hydrodynamic force. Under the assumption of an infinitely short bearing (i.e., $L/(2r) < 0.5$), we derive an expression for F from the one-dimensional Reynolds equation

$$\frac{\partial}{\partial z}\left(h^3 \frac{\partial p}{\partial z}\right) = 6\mu\omega\frac{\partial h}{\partial \theta} + 12\mu\frac{\partial h}{\partial t} \qquad (3)$$

where p is the pressure field between the journal and pads, h is the fluid film thickness, and μ is the constant fluid viscosity. Specifically, by solving (3), we can obtain an analytical solution for the pressure field along each pad arc. Then, solving the following integrals along the pad surface area yield the hydrodynamic forces along the x and y directions due to each pad:

$$F_i = \begin{bmatrix} F_{xi} \\ F_{yi} \end{bmatrix} = r\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_{\underline{\theta}_i}^{\bar{\theta}_i} p_i \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} d\theta dz, i = 1,2,3,4 \qquad (4)$$

where F_i, p_i denote the force and pressure from the ith pad, respectively and $\left[\underline{\theta}_i, \bar{\theta}_i\right] \subseteq S_i, \underline{\theta}_i \le \bar{\theta}_i$ denotes the portion of the ith pad arc where the pressure is nonnegative.

Specifically, by solving (3), we can obtain an analytical solution for the pressure field along each pad arc [3]

$$p_i(q,\dot{q},\alpha_i,\dot{\alpha}_i) = \frac{\upsilon_i \sin\theta - \sigma_i\cos\theta}{h^3}(4z^2 - 1) \qquad (5)$$

where

$$\upsilon_i = \frac{3L^2\mu}{4}\left(\omega x_{pi} - 2\dot{y}_{pi}\right), \quad \sigma_i = \frac{3L^2\mu}{4}\left(\omega y_{pi} + 2\dot{x}_{pi}\right) \qquad (6)$$

$$x_{pi} = x - r\alpha_i \sin(i-1)\frac{\pi}{2}, \quad y_{pi} = y + r\alpha_i\cos(i-1)\frac{\pi}{2} \qquad (7)$$

$$h_i = c - x_{pi}\cos\theta - y_{pi}\sin\theta \qquad (8)$$

To calculate the integrals in (4), we follow the procedure of [3] with some minor corrections. After some lengthy calculations, the total hydrodynamic force acting on the journal is given by

$$F = \sum_{i=1}^{4} F_i = \psi(q,\dot{q},\alpha) + G(q,\dot{q},\alpha)\dot{\alpha} \tag{9}$$

where

$$\psi = -\frac{Lr}{3}\sum_{i=1}^{4}\begin{bmatrix} B_i & -A_i \\ C_i & -B_i \end{bmatrix}\left[\frac{\frac{3L^2\mu}{4}\left(\omega x_{pi} - 2\dot{y}\right)}{\sigma_i}\right]_{\underline{\theta}_i}^{\bar{\theta}_i}, \tag{10}$$

$$G = -\frac{2Lr^2}{3}\cos(i-1)\frac{\pi}{2}\begin{bmatrix} B_1 & B_2 & B_3 & B_4 \\ C_1 & C_2 & C_3 & C_4 \end{bmatrix}_{\underline{\theta}_i}^{\bar{\theta}_i}, \tag{11}$$

A_i, B_i, C_i are nonlinear functions of $q, \dot{q}, \alpha$.

Based on (9), the equations of motion (1) can be expressed as follows

$$m\ddot{q} = \psi(q,\dot{q},\alpha) + W + G(q,\dot{q},\alpha)u \tag{12}$$

where $u = \dot{\alpha}$ is the control input. We consider a horizontal rotor shaft such that $W = (0, -W_0)$, where $W_0 > 0$ denotes the rotor weight. We also make the following assumptions regarding the above controlled dynamics: A1) A fluid film is always present between the journal and pads, i.e., $h_i(q,\alpha_i) > 0$ in (10) $\forall q, \alpha_i$, $i = 1,2,3,4$; A2) the variable α is bounded for all time; and A3) the input matrix G has full row rank for $\forall q, \alpha_i$.

Our control objective is to design a model-based feedback law for u such that $q(t), \dot{q}(t) \to 0$ as $t \to \infty$, under the assumption that $q, \dot{q}$, and α are measurable. To this end, we rewrite (12) in terms of the variable $s = \dot{q} + K_1 q$ to obtain

$$m\dot{s} = \psi(q,\dot{q},\alpha) + W + G(q,\alpha)u + mK_1\dot{q}. \tag{13}$$

From (13), we propose the following feedback linearization-type control law

$$u = G^T(GG^T)^{-1}\left(-\psi - W - mK_1\dot{q} - mK_2 s\right) \tag{14}$$

where $K_1, K_2 \in \Re^{2\times 2}$ are constant, symmetric, and positive definite. The above input yields the exponentially stable, linear closed-loop system $\dot{s} = K_1 s$, i.e., $s(t) \to 0$ as $t \to \infty$ exponentially fast, and therefore $q(t), \dot{q}(t) \to 0$ as $t \to \infty$.

The proposed active tilting-pad bearing under control of the feedback law in (14) was simulated in comparison to the bearing's standard passive operation. When simulating (1), the hydrodynamic force was obtained by solving *full* Reynolds equation. Figure 2 shows that after approximately t = 1.6 sec the journal for the passive bearing reaches a limit cycle about an eccentricity ratio of 0.15 on the negative y axis (i.e., direction of load). On the other hand, the active control is able to prevent the limit cycle and make the journal run concentrically after approximately 5 msec. We note that the control inputs of the active bearing eventually become zero once the journal reaches the origin. That is, the control turns off and the active bearing effectively becomes a passive bearing in the steady state.

ACKNOWLEDGMENTS

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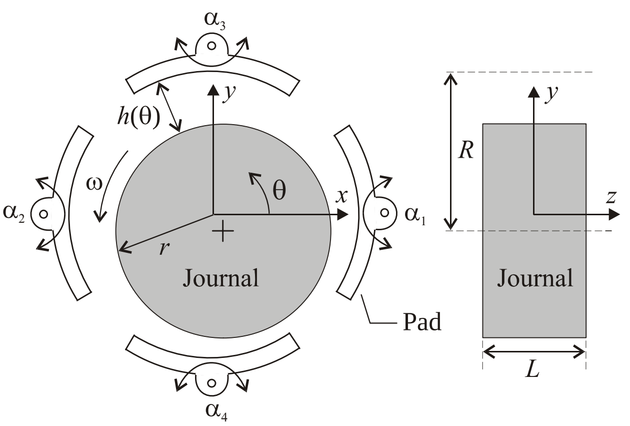

Figure 1: The tilting-pad bearing system.

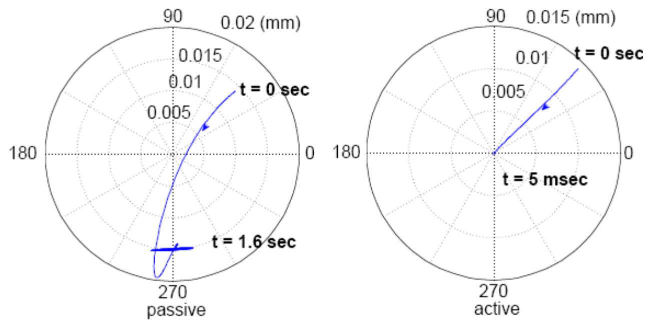

Figure 2: Orbit of the journal center q.