3D MULTI-AGENT FORMATION CONTROL WITH RIGID BODY MANEUVERS

Pengpeng Zhang  
Ph.D. Candidate

Faculty Advisor: Marcio de Queiroz

ABSTRACT

A network of mobile physical entities, such as autonomous vehicles, that collectively perform a complex task beyond their individual capabilities is commonly referred to as a multi-agent system. Numerous multi-agent coordination and cooperation problems have been the subject of considerable attention from researchers over the past several years: aggregation, consensus, social foraging, flocking, synchronization, etc. [1]. In this paper, we are primarily interested in the formation problem. Formation control refers to the behavior where a group of agents acquire and maintain a prescribed geometric shape in space. This paper is devoted to a variant of this problem called formation maneuvering, where agents are required to simultaneously acquire a formation and move cohesively as a virtual rigid body with a pre-defined velocity.

The formation control problem is easy to solve if the agents’ global coordinates can be measured via, for example, a global positioning system (GPS). Unfortunately, GPS has limited accuracy when there is no or limited line of sight between the GPS receiver and satellites. This issue brings up the more challenging problem where each agent has only locally-sensed information about other agents from onboard sensors (e.g., inertial-type navigation system, laser range finder, camera, and/or compass).

When modeling the multi-agent formation shape, graph theory is a convenient tool. A subset of this theory — rigid graph theory — naturally ensures that the inter-agent distance constraints of the desired formation are enforced through the rigidity of the underlying graph. This implies that collisions between agents are avoided while acquiring the formation. Another benefit of employing the inter-agent distances as the controlled variables is that position measurements in a global coordinate frame are not required [2]. An overview of rigid graph theory and its application to formation control of autonomous vehicles can be found in [3].

Previous formation maneuvering results only considered the translational component of the rigid body motion for the swarm of agents. In this paper, we propose a new control law for the formation maneuvering problem that allows for full rigid body motions, i.e., the swarm maneuvers include translation and rotation of the virtual rigid-body. We consider 3D formations where the desired formation is modeled by an infinitesimally and minimally rigid, undirected graph. To achieve the rigid body rotation, we use the leader-follower concept where the leader serves as the axis of rotation. The graph that models the desired formation is constructed such that the leader is in the convex hull of the followers. The motion of the agents is modeled by single-integrator equations. The control is based on the graph rigidity matrix and exploits its special structure to decouple the formation acquisition stability analysis from the formation maneuvering analysis. Using Lyapunov-based arguments, we show that the control ensures exponential formation acquisition and asymptotic convergence of the agent velocities to the desired rigid motion maneuver.

The actual system is represented by a framework \( F = (G, q) \), where \( G = (V, E) \) is an undirected graph, \( V \) is the vertex, \( E \) is the edge set, and \( q = [q_1, \ldots, q_n] \in \mathbb{R}^n \) is the actual coordinate of all the vertices. The desired framework is \( F' = (G, q') \), where \( q' = [q'_1, \ldots, q'_n] \in \mathbb{R}^n \) is the desired location of vertices. The edge function \( \phi(q) \) is given by

\[
\phi(q) = \ldots ||q_i - q_j||^2 \ldots, \ (i, j) \in E
\]  

while the rigidity matrix \( R \) is defined as

\[
R(q) = \frac{1}{2} \frac{\partial \phi(q)}{\partial q}. 
\]  

We consider a system of \( n \) kinematic agents in space modeled by the single integrator

\[
\dot{q}_i = u_i, i = 1, \ldots, n, 
\]  

where \( u_i \in \mathbb{R}^3 \) is the velocity-level control input of the \( i \)th agent. And for formation maneuvering problem, the primary control objective is to design \( u_i \) such that

\[
||q_i(t) - q_j(t)|| \rightarrow d_0 \ as \ t \rightarrow \infty, \ i, j \in V, 
\]
where \( d_{ij} \) is the constant desired distance, which means all the inter-agent distance will converge to the desired distance. The secondary objective is given by

\[
\dot{q}_i(t) - v_{d}(t) \rightarrow 0 \quad \text{as} \quad t \rightarrow \infty, \quad i = 1, \ldots, n, \tag{5}
\]

where \( v_{d} \) is defined in (10), which says the velocity of all agents will track the desired velocity.

The distance tracking error is defined as

\[
e_{ij} = \| \tilde{q}_{ij} \| - d_{ij} \tag{6}
\]

where \( \tilde{q}_{ij} \) is the relative distance between two agents \( i \) and \( j \).

Then the formation maneuvering control law is given by the following theorem.

**Theorem:** Given the formation \( F(t) = (G^*, q(t)) \) let the initial conditions be such that \( e(0) \in \Omega_1 \cap \Omega_2 \) where

\[
\Omega = \{ e \in \mathbb{R}^l \mid \Psi(F, F^*) \leq \delta \}
\]

\[
\Omega = \{ e \in \mathbb{R}^l \mid \text{dist}(q, \text{Iso}(F^*)) < \text{dist}(q, \text{Amb}(F^*)) \},
\]

\( \delta \) is a sufficiently small positive constant and function \( \Psi \) and dist are defined in [4]. Then, the control

\[
u = -kr^T(q)z + v_{d}, \tag{8}
\]

where \( k > 0, z = [z_1, \ldots, z_n] \in \mathbb{R}^l, (i, j) \in E \) with

\[
z_{ij} = ||\tilde{q}_{ij}||^2 - d_{ij}^2, \quad (i, j) \in E,
\]

and \( v_{d} = [v_{d1}, \ldots, v_{dn}] \in \|3\| \) is the desired rigid body velocity specified by

\[
v_{d} = v_i + \omega \times \tilde{q}_{im}, \quad i = 1, \ldots, n, \tag{10}
\]

\( v_i(t) \in \mathbb{R}^l \) denotes the desired translation velocity for the formation, and \( \omega(t) \in \mathbb{R}^3 \) is the desired angular velocity, renders \( e = 0 \) exponentially stable and ensures that (4) and (5) are satisfied.

\[v_i = [1,1,\cos(t)]^T \quad \text{and} \quad \omega = [1,1,1]^T. \tag{11}\]

Figure 2 shows snapshots of the actual formation \( F(t) \) over time. As the agents acquire the desired formation by \( t = 3 \), they maneuver as a rigid body according to the predefined velocity.

**REFERENCES**