Home Work Solutions: HW # 1

2.21 What is the pressure at the bottom of a 5 m tall column of fluid with atmospheric pressure 101 kPa on the top surface if the fluid is
a) water at 20°C          b) glycerine 25°C          or          c) light oil

Solution:

Table A.4: \( \rho_{\text{H}_2\text{O}} = 997 \text{ kg/m}^3 \); \( \rho_{\text{glyc}} = 1260 \text{ kg/m}^3 \); \( \rho_{\text{oil}} = 910 \text{ kg/m}^3 \)

\[ \Delta P = \rho gh \quad P = P_{\text{top}} + \Delta P \]

a) \( \Delta P = \rho gh = 997 \times 9.807 \times 5 = 48887.9 \text{ Pa} \)
   \[ P = 101 + 48.99 = 149.9 \text{ kPa} \]

b) \( \Delta P = \rho gh = 1260 \times 9.807 \times 5 = 61784 \text{ Pa} \)
   \[ P = 101 + 61.8 = 162.8 \text{ kPa} \]

c) \( \Delta P = \rho gh = 910 \times 9.807 \times 5 = 44622 \text{ Pa} \)
   \[ P = 101 + 44.6 = 145.6 \text{ kPa} \]

2.28 A submarine maintains 101 kPa inside it and it dives 240 m down in the ocean having an average density of 1030 kg/m\(^3\). What is the pressure difference between the inside and the outside of the submarine hull?

Solution:

Assume inside is 101 kPa then \( \Delta P \) is from the 240 m column water.
\[ \Delta P = \rho L g = (1030 \times 240 \times 9.807) / 1000 = 2424 \text{ kPa} \]

2.31 The difference in height between the columns of a manometer is 200 mm with a fluid of density 900 kg/m\(^3\). What is the pressure difference? What is the height difference if the same pressure difference is measured using mercury, density 13600 kg/m\(^3\), as manometer fluid?

Solution:

\[ \Delta P = \rho_1 gh_1 = 900 \times 9.807 \times 0.2 = 1765.26 \text{ Pa} = 1.77 \text{ kPa} \]

\[ h_{\text{hg}} = \frac{\Delta P}{(\rho_{\text{hg}} g)} = \frac{(\rho_1 gh_1)}{(\rho_{\text{hg}} g)} = \frac{900}{13600} \times 0.2 = 0.0132 \text{ m} = 13.2 \text{ mm} \]
2.39 A piece of experimental apparatus is located where $g = 9.5 \text{ m/s}^2$ and the temperature is 5°C. An air flow inside the apparatus is determined by measuring the pressure drop across an orifice with a mercury manometer (see Problem 2.27 for density) showing a height difference of 200 mm. What is the pressure drop in kPa?

Solution:

\[
\Delta P = \rho gh \quad \text{and} \quad \rho_{\text{Hg}} = 13600 \text{ kg/m}^3
\]

\[
\Delta P = 13600 \times 9.5 \times 0.2 = 25840 \text{ Pa} = 25.84 \text{ kPa}
\]

2.41 A dam retains a lake 6 m deep. To construct a gate in the dam we need to know the net horizontal force on a 5 m wide and 6 m tall port section that then replaces a 5 m section of the dam. Find the net horizontal force from the water on one side and air on the other side of the port.

Solution:

\[
P_{\text{bot}} = P_0 + \Delta P
\]

\[
\Delta P = \rho gh = 997 \times 9.807 \times 6 = 58665 \text{ Pa} = 58.66 \text{ kPa}
\]

Neglect $\Delta P$ in air

\[
F_{\text{net}} = F_{\text{right}} - F_{\text{left}} = P_{\text{avg}} A - P_0 A
\]

\[
P_{\text{avg}} = P_0 + 0.5 \Delta P \quad \text{Since a linear pressure variation with depth.}
\]

\[
F_{\text{net}} = (P_0 + 0.5 \Delta P)A - P_0 A = 0.5 \Delta P A = 0.5 \times 58.66 \times 5 \times 6 = 880 \text{ kN}
\]
3.18 Find the phase, quality \( x \) if applicable and the missing property \( P \) or \( T \).

Solution:

a. \( H_2O \quad T = 120^\circ C \quad v = 0.5 \text{ m}^3/\text{kg} \)

Table B.1.1 at given \( T \): \( v < v_g = 0.89186 \)

sat. liq. + vap. \( P = P_{sat} = 198.5 \text{ kPa} \),
\[ x = (v - v_l)/v_f = (0.5 - 0.00106)/0.8908 = 0.56 \]

b. \( H_2O \quad P = 100 \text{ kPa} \quad v = 1.8 \text{ m}^3/\text{kg} \)

Table B.1.2 at given \( P \): \( v > v_g = 1.694 \)

sup. vap., interpolate in Table B.1.3
\[ T = \frac{1.8 - 1.694}{1.93636 - 1.694} (150 - 99.62) + 99.62 = 121.65^\circ C \]

c. \( H_2O \quad T = 263 \text{ K} \quad v = 0.2 \text{ m}^3/\text{kg} \)

Table B.1.5 at given \( T = -10^\circ C \): \( v < v_g = 466.757 \)

sat. solid + vap. \( P = P_{sat} = 0.26 \text{ kPa} \),
\[ x = (v - v_l)/v_f = (200 - 0.001)/466.756 = 0.4285 \]

d. \( NH_3 \quad P = 800 \text{ kPa} \quad v = 0.2 \text{ m}^3/\text{kg} \)

Superheated Vapor (\( v > v_g \) at 800 kPa)

Table B 2.2 interpolate between 70\(^\circ\)C and 80\(^\circ\)C
\( T = 71.4^\circ C \)

e. \( NH_3 \quad T = 20^\circ C \quad v = 0.1 \text{ m}^3/\text{kg} \)

Table B.2.1 at given \( T \): \( v < v_g = 0.14922 \)

sat. liq. + vap. \( P = P_{sat} = 857.5 \text{ kPa} \),
\[ x = (v - v_l)/v_f = (0.1 - 0.00164)/0.14758 = 0.666 \]

States shown are placed relative to the two-phase region, not to each other.
3.22 Two tanks are connected as shown in Fig. P3.22, both containing water. Tank A is at 200 kPa, $v = 0.5 \text{ m}^3/\text{kg}$, $V_A = 1 \text{ m}^3$ and tank B contains 3.5 kg at 0.5 MPa, 400°C. The valve is now opened and the two come to a uniform state. Find the final specific volume.

Solution:

Control volume: both tanks. Constant total volume and mass process.

State A1: \((P, v)\) \quad m_A = \frac{V_A}{v_A} = \frac{1}{0.5} = 2 \text{ kg}

State B1: \((P, T)\) \quad \text{Table B.1.3} \quad v_B = 0.6173 \text{ m}^3/\text{kg}

\[ \Rightarrow V_B = m_B v_B = 3.5 \times 0.6173 = 2.1606 \text{ m}^3 \]

Final state: \quad m_{tot} = m_A + m_B = 5.5 \text{ kg}

\[ V_{tot} = V_A + V_B = 3.1606 \text{ m}^3 \]

\[ v_2 = \frac{V_{tot}}{m_{tot}} = 0.5746 \text{ m}^3/\text{kg} \]
3.26 Saturated liquid water at 60°C is put under pressure to decrease the volume by 1% keeping the temperature constant. To what pressure should it be compressed?

Solution:

State 1: \( T = 60°C, \ x = 0.0; \) Table B.1.1: \( v = 0.001017 \text{ m}^3/\text{kg} \)
Process: \( T = \text{constant} = 60°C \)
State 2: \( T, v = 0.99 \times v_f(60°C) = 0.99 \times 0.001017 = 0.0010068 \text{ m}^3/\text{kg} \)

Between 20 \& 30 \text{ MPa} in Table B.1.4, \( P \approx 23.8 \text{ MPa} \)
Consider two tanks, A and B, connected by a valve, as shown in Fig. P3.29. Each has a volume of 200 L and tank A has R-12 at 25°C, 10% liquid and 90% vapor by volume, while tank B is evacuated. The valve is now opened and saturated vapor flows from A to B until the pressure in B has reached that in A, at which point the valve is closed. This process occurs slowly such that all temperatures stay at 25°C throughout the process. How much has the quality changed in tank A during the process?

Solution:

![Diagram of two tanks connected by a valve]

State A1: Table B.3.1 \( v_f = 0.000763, \ v_g = 0.026854 \)

\[
m_{A1} = \frac{V_{\text{liq}}}{v_{f25^\circ C}} + \frac{V_{\text{vap}}}{v_{g25^\circ C}} = 0.1 \times 0.2 + 0.9 \times 0.2 = 26.212 + 6.703 = 32.915 \text{ kg}
\]

\[
x_{A1} = \frac{6.703}{32.915} = 0.2036
\]

State B2: Assume A still two-phase so saturated P for given T

\[
m_{B2} = \frac{V_B}{v_{g25^\circ C}} = 0.2 \times 0.26854 = 7.448 \text{ kg}
\]

State A2: mass left is \( m_{A2} = 32.915 - 7.448 = 25.467 \text{ kg} \)

\[
v_{A2} = \frac{0.2}{25.467} = 0.007853 = 0.000763 + x_{A2} \times 0.026091
\]

\[
x_{A2} = 0.2718 \quad \Delta x = 6.82\%
\]
For a certain experiment, R-22 vapor is contained in a sealed glass tube at 20°C. It is desired to know the pressure at this condition, but there is no means of measuring it, since the tube is sealed. However, if the tube is cooled to -20°C small droplets of liquid are observed on the glass walls. What is the initial pressure?

Solution:

Control volume: R-22 fixed volume (V) & mass (m) at 20°C
Process: cool to -20°C at constant v, so we assume saturated vapor

State 2: \( v = \text{const} = v_g \text{ at } -20°C = 0.092843 \text{ m}^3/\text{kg} \)

State 1: 20°C, 0.092843 m³/kg
   \[ \text{interpolate between 250 and 300 kPa in Table B.4.2} \]
   \[ \Rightarrow \quad P = 291 \text{ kPa} \]