3.49 A cylinder has a thick piston initially held by a pin as shown in Fig. P3.49. The cylinder contains carbon dioxide at 200 kPa and ambient temperature of 290 K. The metal piston has a density of 8000 kg/m³ and the atmospheric pressure is 101 kPa. The pin is now removed, allowing the piston to move and after a while the gas returns to ambient temperature. Is the piston against the stops?

Solution:

Force balance on piston determines equilibrium float pressure.

\[ P_\text{ext on CO}_2 = P_0 + \frac{m_p g}{A_p} = 101 + \frac{A_p \times 0.1 \times 9.807 \times 8000}{A_p \times 1000} = 108.8 \text{ kPa} \]

Pin released, as \( P_1 > P_\text{ext} \) piston moves up, \( T_2 = T_0 \) & if piston at stops,

then \( V_2 = V_1 \times \frac{H_2}{H_1} = V_1 \times 150 / 100 \)

Ideal gas with \( T_2 = T_1 \) then gives

\[ \Rightarrow P_2 = P_1 \times \frac{V_1}{V_2} = 200 \times \frac{100}{150} = 133 \text{ kPa} > P_\text{ext} \]

\[ \Rightarrow \text{piston is at stops, and } P_2 = 133 \text{ kPa} \]
3.55 A piston/cylinder arrangement, shown in Fig. P3.55, contains air at 250 kPa, 300°C. The 50-kg piston has a diameter of 0.1 m and initially pushes against the stops. The atmosphere is at 100 kPa and 20°C. The cylinder now cools as heat is transferred to the ambient.

a. At what temperature does the piston begin to move down?

b. How far has the piston dropped when the temperature reaches ambient?

Solution:

\[
P_{\text{float}} = P_o \left(1 + \frac{m_p g}{A_p \rho}\right) = 100 + \frac{50 \times 9.807}{0.00785 \times 1000} = 162.5 \text{ kPa} = P_2 = P_3
\]

To find temperature at 2 assume ideal gas:

\[
T_2 = T_1 \times \frac{P_2}{P_1} = 573.15 \times \frac{162.5}{250} = 372.5 \text{ K}
\]

b) Process 2 -> 3 is constant pressure as piston floats to \(T_3 = T_o = 293.15 \text{ K}\)

\[
V_2 = V_1 = A_p \times h = 0.00785 \times 0.25 = 0.00196 \text{ m}^3 = 1.96 \text{ L}
\]

Ideal gas and \(P_2 = P_3\) \(\Rightarrow\)

\[
V_3 = V_2 \times \frac{T_3}{T_2} = 1.96 \times \frac{293.15}{372.5} = 1.54 \text{ L}
\]

\[
\Delta H = (V_2 - V_3) / A = (1.96 - 1.54) \times \frac{0.001}{0.00785} = 0.053 \text{ m} = 5.3 \text{ cm}
\]
3.61 Air in a tank is at 1 MPa and room temperature of 20°C. It is used to fill an initially empty balloon to a pressure of 200 kPa, at which point the diameter is 2 m and the temperature is 20°C. Assume the pressure in the balloon is linearly proportional to its diameter and that the air in the tank also remains at 20°C throughout the process. Find the mass of air in the balloon and the minimum required volume of the tank.

Solution: Assume air is an ideal gas.

Balloon final state: \( V_2 = (\frac{4}{3}) \pi r^3 = (\frac{4}{3}) \pi 2^3 = 33.51 \text{ m}^3 \)

\[ m_{2\text{bal}} = \frac{P_2}{R} \frac{V_2}{T_2} = 200 \times 33.51 / 0.287 \times 293.15 = 79.66 \text{ kg} \]

Tank must have \( P_2 \geq 200 \text{ kPa} \) \( \Rightarrow m_{2\text{tank}} \geq \frac{P_2}{RT_2} V_{\text{TANK}} \)

Initial mass must be enough:

\[ m_1 = m_{2\text{bal}} + m_{2\text{tank}} = \frac{P_1}{R} \frac{V_1}{T_1} \]

\[ P_1 \frac{V_{\text{TANK}}}{RT_1} = m_{2\text{bal}} + P_2 \frac{V_{\text{TANK}}}{RT_2} \]

\[ V_{\text{TANK}} = RT \frac{m_{2\text{bal}}}{(P_1 - P_2)} = 0.287 \times 293.15 \times 79.66 / (1000 - 200) \]

\[ = 8.377 \text{ m}^3 \]
4.13 A vertical cylinder (Fig. P4.13) has a 90-kg piston locked with a pin trapping 10 L of R-22 at 10°C, 90% quality inside. Atmospheric pressure is 100 kPa, and the cylinder cross-sectional area is 0.006 m². The pin is removed, allowing the piston to move and come to rest with a final temperature of 10°C for the R-22. Find the final pressure, final volume and the work done by the R-22.

Solution:

State 1: \((T,x)\) from table B.4.1
\[
\begin{align*}
v_1 &= 0.0008 + 0.9 \times 0.03391 = 0.03132 \\
m &= V_1/v_1 = 0.010/0.03132 = 0.319 \text{ kg}
\end{align*}
\]
Force balance on piston gives the equilibrium pressure
\[
P_2 = P_0 + \frac{m_p g}{A_p} = 100 + \frac{90 \times 9.807}{0.006 \times 1000} = 247 \text{ kPa}
\]

State 2: \((T,P)\) interpolate \(V_2 = mv_2 = 0.319 \times 0.10565 = 0.0337 \text{ m}^3 = 33.7 \text{ L}\)
\[
W_2 = \int P_{\text{equil}} \, dV = P_2(V_2-V_1) = 247(0.0337-0.010) = 5.85 \text{ kJ}
\]
A piston cylinder contains 0.5 kg air at 500 kPa, 500 K. The air expands in a process so \( P \) is linearly decreasing with volume to a final state of 100 kPa, 300 K. Find the work in the process.

Solution:

Process: \( P = A + B V \)  \( \text{ (linear in } V, \text{ decreasing means } B \text{ is negative) } \)

From the process:

\[
1 W_2 = \int PdV = \text{AREA} = \frac{1}{2} (P_1 + P_2)(V_2 - V_1)
\]

\[
V_1 = mR \frac{T_1}{P_1} = 0.5 \times 0.287 \times (500/500) = 0.1435 \text{ m}^3
\]

\[
V_2 = mR \frac{T_2}{P_2} = 0.5 \times 0.287 \times (300/100) = 0.4305 \text{ m}^3
\]

\[
1 W_2 = \frac{1}{2} \times (500+100) \times (0.4305-0.1435) = 86.1 \text{ kJ}
\]
A balloon behaves such that the pressure inside is proportional to the diameter squared. It contains 2 kg of ammonia at 0°C, 60% quality. The balloon and ammonia are now heated so that a final pressure of 600 kPa is reached. Considering the ammonia as a control mass, find the amount of work done in the process.

Solution:

Process: \( P \propto D^2 \), with \( V \propto D^3 \) this implies \( P \propto D^2 \propto V^{2/3} \) so \( PV^{-2/3} = \text{constant} \), which is a polytropic process, \( n = -2/3 \)

From table B.2.1: \( V_1 = mv_1 = 2(0.001566 + 0.6 \times 0.28783) = 0.3485 \text{ m}^3 \)

\[
V_2 = V_1 \left( \frac{P_2}{P_1} \right)^{3/2} = 0.3485 \left( \frac{600}{429.3} \right)^{3/2} = 0.5758 \text{ m}^3
\]

\[
W_2 = \int P \, dV = \frac{P_2 V_2 - P_1 V_1}{1 - n} \quad \text{(Equation 4.4)}
\]

\[
= \frac{600 \times 0.5758 - 429.3 \times 0.3485}{1 - (-2/3)} = 117.5 \text{ kJ}
\]
10 kg of water in a piston cylinder arrangement exists as saturated liquid/vapor at 100 kPa, with a quality of 50%. It is now heated so the volume triples. The mass of the piston is such that a cylinder pressure of 200 kPa will float it.

(a) Find the final temperature and volume of the water.

(b) Find the work given out by the water.

Solution:

Take CV as the water \( m_2 = m_1 = m \);

Process: \( v = \text{constant until } P = P_{\text{lift}} \), then \( P \) is constant.

State 1: \( v_1 = 0.001043 + 0.5 \times 1.69296 = 0.8475 \)

State 2: \( v_2, P_2 \leq P_{\text{lift}} \Rightarrow v_2 = 3 \times 0.8475 = 2.5425 \);

\( T_2 = 829°C \quad V_2 = m v_2 = 25.425 \text{ m}^3 \)

\[
W_2 = \int P \, dV = P_{\text{lift}} \times (V_2 - V_1)
\]

\[
= 200 \times 10 \times (2.5425 - 0.8475) = 3390 \text{ kJ}
\]
A cylinder containing 1 kg of ammonia has an externally loaded piston. Initially the ammonia is at 2 MPa, 180°C and is now cooled to saturated vapor at 40°C, and then further cooled to 20°C, at which point the quality is 50%. Find the total work for the process, assuming a piecewise linear variation of $P$ versus $V$.

Solution:

State 1: (T, P)  Table B.2.2  
$v_1 = 0.10571$

State 2: (T, x)  Table B.2.1 sat. vap.  
$P_2 = 1555$ kPa,  $v_2 = 0.08313$

State 3: (T, x)  $P_3 = 857$ kPa,  $v_3 = (0.001638 + 0.14922)/2 = 0.07543$

Sum the the work as two integrals each evaluated by the area in the P-v diagram.

$$W_3 = \int_{1}^{3} PdV \approx \left( \frac{P_1 + P_2}{2} \right) m(v_2 - v_1) + \left( \frac{P_2 + P_3}{2} \right) m(v_3 - v_2)$$

$$= \frac{2000 + 1555}{2} \cdot \frac{1(0.08313 - 0.10571)}{1} + \frac{1555 + 857}{2} \cdot \frac{1(0.07543 - 0.08313)}{1}$$

$$= -49.4 \text{ kJ}$$
4.60 A pot of steel, conductivity $50 \text{ W/m K}$, with a $5 \text{ mm}$ thick bottom is filled with $15^\circ \text{C}$ liquid water. The pot has a diameter of $20 \text{ cm}$ and is now placed on an electric stove that delivers $250 \text{ W}$ as heat transfer. Find the temperature on the outer pot bottom surface assuming the inner surface is at $15^\circ \text{C}$.

Solution:

Steady conduction through the bottom of the steel pot. Assume the inside surface is at the liquid water temperature.

$$\dot{Q} = k \ A \ \frac{\Delta T}{\Delta x} \Rightarrow \Delta T = \frac{\dot{Q} \ \Delta x}{kA}$$

$$\Delta T = 250 \times 0.005/(50 \times \frac{\pi}{4} \times 0.2^2) = 0.796$$

$$T = 15 + 0.796 \approx 15.8^\circ \text{C}$$

4.65 Due to a faulty door contact the small light bulb ($25 \text{ W}$) inside a refrigerator is kept on and limited insulation lets $50 \text{ W}$ of energy from the outside seep into the refrigerated space. How much of a temperature difference to the ambient at $20^\circ \text{C}$ must the refrigerator have in its heat exchanger with an area of $1 \text{ m}^2$ and an average heat transfer coefficient of $15 \text{ W/m}^2 \text{ K}$ to reject the leaks of energy.

Solution:

$$\dot{Q}_{\text{tot}} = 25 + 50 = 75 \text{ W to go out}$$

$$\dot{Q} = hA\Delta T = 15 \times 1 \times \Delta T = 75$$

$$\Delta T = \frac{\dot{Q}}{hA} = 75/(15 \times 1) = 5^\circ \text{C}$$

Or T must be at least $25^\circ \text{C}$