Homework # 3 Solutions

5.36  A 1-L capsule of water at 700 kPa, 150°C is placed in a larger insulated and otherwise evacuated vessel. The capsule breaks and its contents fill the entire volume. If the final pressure should not exceed 125 kPa, what should the vessel volume be?

Solution:

C.V. Larger vessel.

Continuity: \( m_2 = m_1 = m = \frac{V}{v_1} = 0.916 \text{ kg} \)

Process: expansion with \( Q_2 = 0, \ W_2 = 0 \)

Energy: \( m(u_2 - u_1) = Q_2 - W_2 = u_2 - u_1 \)

State 1: \( v_1 = v_f = 0.001091 \text{ m}^3/\text{kg}; \quad u_1 = u_f = 631.66 \text{ kJ/kg} \)

State 2: \( P_2, u_2 \Rightarrow x_2 = \frac{(631.66 - 444.16)/2069.3 = 0.09061}{0.001048 + 0.09061 \cdot 1.37385 = 0.1255 \text{ m}^3/\text{kg}} \)

\( V_2 = m v_2 = 0.916 \times 0.1255 = 0.115 \text{ m}^3 = 115 \text{ L} \)
A vertical cylinder fitted with a piston contains 5 kg of R-22 at 10°C, shown in Fig. P5.40. Heat is transferred to the system, causing the piston to rise until it reaches a set of stops at which point the volume has doubled. Additional heat is transferred until the temperature inside reaches 50°C, at which point the pressure inside the cylinder is 1.3 MPa.

a. What is the quality at the initial state?
b. Calculate the heat transfer for the overall process.

Solution:

C.V. R-22. Control mass goes through process: 1 -> 2 -> 3

As piston floats pressure is constant (1 -> 2) and the volume is constant for the second part (2 -> 3). So we have: \( v_3 = v_2 = 2 \times v_1 \)

State 3: Table B.4.2 (P,T) \( v_3 = 0.02015 \)

\[ u_3 = h - Pv = 274.39 - 1300 \times 0.02015 = 248.2 \text{ kJ/kg} \]

\[ v_1 = 0.010075 = 0.0008 + x_1 \times 0.03391 \quad \Rightarrow \quad x_1 = 0.2735 \]

b) \( u_1 = 55.92 + 0.271 \times 173.87 = 103.5 \)

State 2: \( v_2 = 0.02015 \), \( P_2 = P_1 = 681 \text{ kPa} \) this is still 2-phase.

\[ 1 W_1 = \int_1^2 PdV = P_1(V_2 - V_1) = 681 \times 5 (0.02 - 0.01) = 34.1 \text{ kJ} \]

\[ 1 Q_1 = m(u_3-u_1) + 1 W_1 = 5(248.2-103.5) + 34.1 = 757.6 \text{ kJ} \]
5.59 A 25 kg steel tank initially at $-10^\circ$C is filled up with 100 kg of milk (assume properties as water) at 30$^\circ$C. The milk and the steel come to a uniform temperature of +5 $^\circ$C in a storage room. How much heat transfer is needed for this process?

Solution

C.V. Steel + Milk. This is a control mass.

Energy Eq.5.11: \[ U_2 - U_1 = \dot{Q}_2 - \dot{W}_2 = \dot{Q}_2 \]

Process: \( V = \text{constant} \), so there is no work \( \dot{W}_2 = 0 \).

Use Eq.5.18 and values from A.3 and A.4 to evaluate changes in \( u \)

\[ \dot{Q}_2 = m_{\text{steel}}(u_2 - u_1)_{\text{steel}} + m_{\text{milk}}(u_2 - u_1)_{\text{milk}} \]

\[ = 25 \times 0.46 \times [5 - (-10)] + 100 \times 4.18 \times (5 - 30) \]

\[ = 172.5 - 10450 = -10277 \text{ kJ} \]
5.75 Air in a piston/cylinder at 200 kPa, 600 K, is expanded in a constant-pressure process to twice the initial volume (state 2), shown in Fig. P5.69. The piston is then locked with a pin and heat is transferred to a final temperature of 600 K. Find $P$, $T$, and $h$ for states 2 and 3, and find the work and heat transfer in both processes.

Solution:

C.V. Air. Control mass $m_2 = m_3 = m_1$

Energy Eq.5.11: $u_2 - u_1 = T_1 w_2$.

Process 1 to 2: $P = \text{constant} \Rightarrow T_2 = T_1 \frac{v_2}{v_1} = 2T_1 = 1200 \text{ K}$

Ideal gas $Pv = RT \Rightarrow T_2 = T_1 \frac{v_2}{v_1} = 2T_1 = 1200 \text{ K}$

$P_2 = P_1 = 200 \text{ kPa}, \quad T_2 = \frac{RT_1}{1} = 172.2 \text{ kJ/kg}$

Table A.7 $h_2 = 1277.8 \text{ kJ/kg}, \quad h_3 = h_1 = 607.3 \text{ kJ/kg}$

$1q_2 = u_2 - u_1 + T_1 w_2 = h_2 - h_1 = 1277.8 - 607.3 = 670.5 \text{ kJ/kg}$

Process 2$\rightarrow$3: $v_3 = 2v_1 \Rightarrow 2w_3 = 0$.

$P_3 = P_2 \frac{T_3}{T_2} = P_1 \frac{T_1}{2T_1} = P_1/2 = 100 \text{ kPa}$

$2q_3 = u_3 - u_2 = 435.1 - 933.4 = -498.3 \text{ kJ/kg}$
5.86 An air pistol contains compressed air in a small cylinder, shown in Fig. P5.86. Assume that the volume is 1 cm$^3$, pressure is 1 MPa, and the temperature is 27°C when armed. A bullet, $m = 15$ g, acts as a piston initially held by a pin (trigger); when released, the air expands in an isothermal process ($T = \text{constant}$). If the air pressure is 0.1 MPa in the cylinder as the bullet leaves the gun, find

a. The final volume and the mass of air.
b. The work done by the air and work done on the atmosphere.
c. The work to the bullet and the bullet exit velocity.

Solution:

C.V. Air.

Air ideal gas: \[ m_{\text{air}} = \frac{P_1V_1}{RT_1} = 1000 \times 10^{-6}/(0.287 \times 300) = 1.17 \times 10^{-5} \text{ kg} \]

Process: \[ PV = \text{const} = P_1V_1 = P_2V_2 \implies V_2 = V_1 \frac{P_1}{P_2} = 10 \text{ cm}^3 \]

\[ W_2 = \int PdV = \int \frac{P_1V_1}{V} \, dV = P_1V_1 \ln \left( \frac{V_2}{V_1} \right) = 2.32 \text{ J} \]

\[ W_{2,\text{ATM}} = P_0(V_2 - V_1) = 101 \times (10-1) \times 10^{-6} \text{ kJ} = 0.909 \text{ J} \]

\[ W_{\text{bullet}} = W_2 - W_{2,\text{ATM}} = 1.411 \text{ J} = \frac{1}{2}m_{\text{bullet}}(V_{\text{exit}})^2 \]

\[ V_{\text{exit}} = (2W_{\text{bullet}}/m_B)^{1/2} = (2 \times 1.411/0.015)^{1/2} = 13.72 \text{ m/s} \]
6.11 A sluice gate dams water up 5 m. There is a small hole at the bottom of the gate so liquid water at 20°C comes out of a 1 cm diameter hole. Neglect any changes in internal energy and find the exit velocity and mass flow rate.

Solution:

Energy Eq. 6.13: \[ h_1 + \frac{1}{2} V_1^2 + g Z_1 = h_2 + \frac{1}{2} V_2^2 + g Z_2 \]

Process: \[ h_1 = h_2 \quad \text{both at P = 1 atm} \]
\[ V_1 = 0 \quad Z_1 = Z_2 + 5 \text{ m} \]

\[ \frac{1}{2} V_2^2 = g (Z_1 - Z_2) \]
\[ V_2 = \sqrt{2g(Z_1 - Z_2)} \]
\[ = \sqrt{2 \times 9.806 \times 5} = 9.902 \text{ m/s} \]

\[ \dot{m} = \rho AV = \frac{\pi}{4} D^2 \times (V_2/v) \]
\[ = \frac{\pi}{4} \times (0.01)^2 \times (9.902 / 0.001002) \]
\[ = 0.776 \text{ kg/s} \]
Water at 1.5 MPa, 150°C, is throttled adiabatically through a valve to 200 kPa. The inlet velocity is 5 m/s, and the inlet and exit pipe diameters are the same. Determine the state (neglecting kinetic energy in the energy equation) and the velocity of the water at the exit.

Solution:

CV: valve. \( \dot{m} = \text{const}, \quad A = \text{const} \)

\[ \Rightarrow V_e = V_i \left( \frac{v_e}{v_i} \right) \]

Energy Eq.6.13:

\[ h_i + \frac{1}{2} v_i^2 = \frac{1}{2} v_e^2 + h_e \quad \text{or} \quad (h_e - h_i) + \frac{1}{2} v_i^2 \left[ \left( \frac{v_e}{v_i} \right)^2 - 1 \right] = 0 \]

Now neglect the kinetic energy terms (relatively small) from table B.1.1 we have the compressed liquid approximated with saturated liquid same T

\[ h_e = h_i = 632.18 \text{ kJ/kg} ; \quad v_i = 0.001090 \]

Table B.1.2: \( h_e = 504.68 + x_e \times 2201.96, \)

Substituting and solving, \( x_e = 0.0579 \)

\[ v_e = 0.001061 + x_e \times 0.88467 = 0.052286 \]

\[ V_e = 5 \left( \frac{0.052286}{0.00109} \right) = 240 \text{ m/s} \]
6.25  Hoover Dam across the Colorado River dams up Lake Mead 200 m higher than the river downstream. The electric generators driven by water-powered turbines deliver 1300 MW of power. If the water is 17.5°C, find the minimum amount of water running through the turbines.

Solution:

C.V.: \( \text{H}_2\text{O} \) pipe + turbines,

Continuity: \( \dot{m}_{\text{in}} = \dot{m}_{\text{ex}} \),

Energy Eq.6.13:

\[
(h + \frac{V^2}{2} + gz)_{\text{in}} = (h + \frac{V^2}{2} + gz)_{\text{ex}} + w_T
\]

Water states: \( h_{\text{in}} \equiv h_{\text{ex}} \), \( v_{\text{in}} \equiv v_{\text{ex}} \) so

Now the specific turbine work becomes

\[
w_T = gz_{\text{in}} - gz_{\text{ex}} = 9.807 \times 200/1000 = 1.961 \text{ kJ/kg}
\]

\[
\dot{m} = \frac{\dot{W}_T}{w_T} = \frac{1300 \times 10^3 \text{ kW}}{1.961 \text{ kJ/kg}} = 6.63 \times 10^5 \text{ kg/s}
\]

\[
\dot{V} = \dot{m}v = 6.63 \times 10^5 \times 0.001001 = 664 \text{ m}^3/\text{s}
\]
6.41 The main waterline into a tall building has a pressure of 600 kPa at 5 m below ground level. A pump brings the pressure up so the water can be delivered at 200 kPa at the top floor 150 m above ground level. Assume a flow rate of 10 kg/s liquid water at 10°C and neglect any difference in kinetic energy and internal energy u. Find the pump work.

Solution:

C.V. Pipe from inlet at -5 m up to exit at +150 m, 200 kPa.

Energy Eq.6.13: \[ h_i + \frac{1}{2} V_i^2 + gZ_i = h_e + \frac{1}{2} V_e^2 + gZ_e + w \]

With the same u the difference in h's are the Pv terms

\[ w = h_i - h_e + \frac{1}{2} (V_i^2 - V_e^2) + g (Z_i - Z_e) \]
\[ = P_i V_i - P_e V_e + g (Z_i - Z_e) \]
\[ = 600 \times 0.001 - 200 \times 0.001 + 9.806 \times (-5-150)/1000 \]
\[ = 0.4 - 1.52 = -1.12 \text{ kJ/kg} \]
\[ \dot{W} = \dot{m}w = 10 \times (-1.12) = -11.2 \text{ kW} \]

6.46 A steam turbine receives steam from two boilers. One flow is 5 kg/s at 3 MPa, 700°C and the other flow is 15 kg/s at 800 kPa, 500°C. The exit state is 10 kPa, with a quality of 96%. Find the total power out of the adiabatic turbine.

Solution:

C.V. whole turbine steady, 2 inlets, 1 exit, no heat transfer \( \dot{Q} = 0 \)

Continuity Eq.6.9: \[ \dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 5 + 15 = 20 \text{ kg/s} \]

Energy Eq.6.10: \[ \dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 + \dot{W}_T \]

Table B.1.3: \[ h_1 = 3911.7 \text{ kJ/kg}, \]
\[ h_2 = 3480.6 \text{ kJ/kg} \]

Table B.1.2: \[ h_3 = 191.8 + 0.96 \times 2392.8 \]
\[ = 2488.9 \text{ kJ/kg} \]

\[ \dot{W}_T = 5 \times 3911.7 + 15 \times 3480.6 - 20 \times 2488.9 = 21990 \text{ kW} = 22 \text{ MW} \]