8.31 A heavily-insulated cylinder fitted with a frictionless piston contains ammonia at 5°C, 92.9% quality, at which point the volume is 200 L. The external force on the piston is now increased slowly, compressing the ammonia until its temperature reaches 50°C. How much work is done on the ammonia during this process?

Solution:
C.V. ammonia in cylinder, insulated so assume adiabatic  \( Q = 0 \).

Cont. Eq.: \( m_2 = m_1 = m \);

Energy Eq. 5.11: \( m(u_2 - u_1) = \int_{Q_1}^{Q_2} -W_2 \)

Entropy Eq. 8.3: \( m(s_2 - s_1) = \int dQ/T \)

State 1: \( T_1 = 5\degree C, x_1 = 0.929, V_1 = 200 \text{ L} = 0.2 \text{ m}^3 \)

\( \begin{align*}
\nu_1 &= \nu_1 + x_1 \nu_{fg} = 0.001583 + 0.929 \times 0.2414 = 0.2258 \text{ m}^3/\text{kg}, \\
u_1 &= u_1 + x_1 u_{fg} = 202.8 + 0.929 \times 1119.2 = 1242.5 \text{ kJ/kg} \\
s_1 &= s_f + x_1 s_{fg} = 0.7951 + 0.929 \times 4.44715 = 4.9491 \text{ kJ/kg K} \\
m_1 &= V_1/\nu_1 = 0.2 / 0.2258 = 0.886 \text{ kg} \\
\text{Process: } 1 \rightarrow 2 \text{ Adiabatic } \int_{Q_1}^{Q_2} = 0 \text{ & Reversible } \Rightarrow s_1 = s_2
\end{align*} \)

State 2: \( T_2 = 50\degree C, s_2 = s_1 = 4.9491 \text{ kJ/kg K} \)

superheated vapor, interpolate in Table B.2.2 \( \Rightarrow \) \( P_2 = 1600 \text{ kPa} \), \( u_2 = 1364.9 \text{ kJ/kg} \)

Energy equation gives the work as \( \int_{Q_1}^{Q_2} = m(u_1 - u_2) = 0.886 \left( 1242.5 - 1364.9 \right) = -108.4 \text{ kJ} \)
8.39 An insulated cylinder/piston has an initial volume of 0.15 m$^3$ and contains steam at 400 kPa, 200$^\circ$C. The steam is expanded adiabatically, and the work output is measured very carefully to be 30 kJ. It is claimed that the final state of the water is in the two-phase (liquid and vapor) region. What is your evaluation of the claim?

Solution:

C.V. Water.

Energy Eq. 5.11: \[ m(u_2 - u_1) = \int Q_2 - W_2 \]

Entropy Eq. 8.3: \[ m(s_2 - s_1) = \int \frac{dQ}{T} \]

Process: \( Q_2 = 0 \) and reversible

State 1: (T, P) Table B.1.3

\[ \nu_1 = 0.5342, \quad u_1 = 2646.8, \quad s_1 = 7.1706 \text{ kJ/kg K} \]

\[
\begin{align*}
V_1 & = \frac{V_1}{v_1} = \frac{0.15}{0.5342} = 0.2808 \text{ kg} \\
m & = \frac{V_1}{v_1} = \frac{15}{0.5342} = 28.08 \text{ kg}
\end{align*}
\]

With the assumed reversible process we have from entropy equation

\[ s_2 = s_1 = 7.1706 \text{ kJ/kg K} \]

and from the energy equation

\[ u_2 = u_1 - W_2/m = 2646.8 - \frac{30}{0.2808} = 2540.0 \text{ kJ/kg} \]

State 2 given by \((u, s)\) check Table B.1.1: \( s_G \) (at \( u_G = 2540 \)) = 7.0259 < \( s_1 \)

\[ \Rightarrow \text{ State 2 must be in superheated vapor region.} \]
8.52 Two tanks contain steam, and they are both connected to a piston/cylinder as shown in Fig. P8.52. Initially the piston is at the bottom and the mass of the piston is such that a pressure of 1.4 MPa below it will be able to lift it. Steam in A is 4 kg at 7 MPa, 700°C and B has 2 kg at 3 MPa, 350°C. The two valves are opened, and the water comes to a uniform state. Find the final temperature and the total entropy generation, assuming no heat transfer.

Solution:
Control mass: All water $m_A + m_B$.
Continuity Eq.: $m_2 = m_A + m_B = 6 \text{ kg}$

Energy Eq.5.11: $m_2u_2 - m_Au_{A1} - m_Bu_{B1} = \dot{Q}_2 - \dot{W}_2 = -\dot{W}_2$
Entropy Eq.8.14: $m_2s_2 - m_As_{A1} - m Bs_{B1} = \dot{s}_{\text{gen}}$

B.1: $v_{A1} = 0.06283$, $u_{A1} = 3448.5$, $s_{A1} = 7.3476$, $V_A = 0.2513 \text{ m}^3$
B.1: $v_{B1} = 0.09053$, $u_{B1} = 2843.7$, $s_{B1} = 6.7428$, $V_B = 0.1811 \text{ m}^3$

The only possible $P$, $V$ combinations for state 2 are on the two lines.
Assume $V_2 > V_A + V_B \Rightarrow P_2 = P_{\text{lift}}$, $\dot{W}_2 = P_2(V_2 - V_A - V_B)$

Substitute into energy equation:

$m_2h_2 = m_Au_{A1} + m_Bu_{B1} + P_2(V_A + V_B)$

$= 4 \times 3448.5 + 2 \times 2843.7 + 1400 \times 0.4324$

State 2: $h_2 = 3347.8 \text{ kJ/kg}$, $P_2 = 1400 \text{ kPa}$, $v_2 = 0.2323$, $s_2 = 7.433$

$T_2 = 441.9 \degree C$,

Check assumption: $V_2 = m_2v_2 = 1.394 \text{ m}^3 > V_A + V_B \quad \text{OK}$

$1s_{\text{gen}} = 6 \times 7.433 - 4 \times 7.3476 - 2 \times 6.7428 = 1.722 \text{ kJ/K}$
A piston/cylinder device keeping a constant pressure has 1 kg water at 20°C and 1 kg of water at 100°C both at 500 kPa separated by a thin membrane. The membrane is broken and the water comes to a uniform state with no external heat transfer. Find the final temperature and the entropy generation for the process.

Solution:

Continuity Eq.: \[ m_2 - m_A - m_B = 0 \]

Energy Eq.5.11: \[ m_2 u_2 - m_A u_A - m_B u_B = -\int W_2 \]

Entropy Eq.8.14: \[ m_2 s_2 - m_A s_A - m_B s_B = \int dQ/T + \int S_{2 \text{ gen}} \]

Process: \[ P = \text{Constant} \Rightarrow \int PdV = P(V_2 - V_1) \]

\[ Q = 0 \]

Substitute the work term into the energy equation and rearrange to get

\[ m_2 u_2 + P_2 V_2 = m_2 h_2 = m_A u_A + m_B u_B + P V_1 = m_A h_A + m_B h_B \]

where the last rewrite used \( PV_1 = PV_A + PV_B \).

State A1: Table B.1.4 \( h_A = 84.41 \) \( s_A = 0.2965 \)

State B1: Table B.1.4 \( h_B = 419.32 \) \( s_B = 1.3065 \)

Energy equation gives:

\[ h_2 = \frac{m_A}{m_2} h_A + \frac{m_B}{m_2} h_B = \frac{1}{2} 84.41 + \frac{1}{2} 419.32 = 251.865 \]

State 2: \( h_2 = 251.865 \) \( P_2 = 500 \text{ kPa} \) from Table B.1.4

\( T_2 = 60.085^\circ \text{C} \), \( s_2 = 0.83184 \text{ kJ/kg K} \)

With the zero heat transfer we have

\[ \int S_{2 \text{ gen}} = m_2 s_2 - m_A s_A - m_B s_B \]

\[ = 2 \times 0.83184 - 1 \times 0.2965 - 1 \times 1.3065 = 0.0607 \text{ kJ/K} \]
8.61 Liquid lead initially at 500°C is poured into a form so that it holds 2 kg. It then cools at constant pressure down to room temperature of 20°C as heat is transferred to the room. The melting point of lead is 327°C and the enthalpy change between the phases, \( h_f \), is 24.6 kJ/kg. The specific heat is 0.138 kJ/kg K for the solid and 0.155 kJ/kg K for the liquid. Calculate the net entropy change for this process.

Solution:

C.V. Lead, constant pressure process

\[
m_{pb}(u_2 - u_1)_{pb} = 1Q_2 - P(V_2 - V_1)
\]

\[
1Q_2 = m_{pb}(h_2 - h_1) = m_{pb}(h_2 - h_{327,sol} - h_{f} + h_{327,l} - h_{500})
\]

\[
= 2 \times (0.138 \times (20 - 327) - 24.6 + 0.155 \times (327 - 500))
\]

\[
= -84.732 - 49.2 - 53.63 = -187.56 \text{ kJ}
\]

\[
\Delta S_{CV} = m_{pb}[C_{p,\text{sol}}\ln(T_2/600) - (h_{f}/600) + C_{p,\text{liq}}\ln(600/T_1)]
\]

\[
= 2 \times [0.138 \ln \frac{293.15}{600} - \frac{24.6}{600} + 0.155 \ln \frac{600}{773.15}] = -0.358 \text{ kJ/K}
\]

\[
\Delta S_{SUR} = \frac{1}{T_0} Q_2 = \frac{-187.56}{293.15} = 0.64 \text{ kJ/K}
\]

The net entropy change from Eq. 8.18 is equivalent to total entropy generation

\[
\Delta S_{\text{net}} = \Delta S_{CV} + \Delta S_{SUR} = \textbf{0.282 kJ/K}
\]
8.70 A handheld pump for a bicycle has a volume of 25 cm$^3$ when fully extended. You now press the plunger (piston) in while holding your thumb over the exit hole so that an air pressure of 300 kPa is obtained. The outside atmosphere is at $P_0$, $T_0$. Consider two cases: (1) it is done quickly (~1 s), and (2) it is done very slowly (~1 h).

a. State assumptions about the process for each case.

b. Find the final volume and temperature for both cases.

Solution:

C.V. Air in pump. Assume that both cases result in a reversible process.

State 1: $P_0$, $T_0$  
State 2: 300 kPa, $?$

One piece of information must resolve the $?$ for a state 2 property.

Case I) Quickly means no time for heat transfer

$$ Q = 0, \text{ so a reversible adiabatic compression.} $$

$$ u_2 - u_1 = -1s_2 : \quad s_2 - s_1 = \int \frac{dq}{T} + s_{2\text{gen}} = 0 $$

With constant $s$ and constant heat capacity we use Eq.8.32

$$ T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 298 \left( \frac{300}{101,325} \right)^{\frac{0.4}{1.4}} = 296.08 \times 3^{0.28575} = 405.3 \text{ K} $$

Use ideal gas law $PV = mRT$ at both states so ratio gives

$$ \Rightarrow \quad V_2 = \frac{P_1V_1T_2}{T_1P_2} = 11.48 \text{ cm}^3 $$

Case II) Slowly, time for heat transfer so $T = \text{constant} = T_0$.

The process is then a reversible isothermal compression.

$$ T_2 = T_0 = 298 \text{ K} \quad \Rightarrow \quad V_2 = \frac{V_1P_1}{P_2} = 8.44 \text{ cm}^3 $$
Two rigid tanks each contain 10 kg N\textsubscript{2} gas at 1000 K, 500 kPa. They are now thermally connected to a reversible heat pump, which heats one and cools the other with no heat transfer to the surroundings. When one tank is heated to 1500 K the process stops. Find the final (P, T) in both tanks and the work input to the heat pump, assuming constant heat capacities.

**Solution:**

**Control volume of hot tank B,**

Process = constant volume & mass  so no work

Energy equation Eq.5.11 and specific heat in Eq.5.20 gives

\[ U_2 - U_1 = m C_v (T_2 - T_1) = 1 Q_2 = 10 \times 0.7448 \times 500 = 3724 \text{ kJ} \]

\[ P_2 = P_1 \frac{T_2}{T_1} = 1.5 (P_1) = 750 \text{ kPa} \]

To fix temperature in cold tank, C.V.: total

For this CV only W\textsubscript{HP} cross the control surface no heat transfer. The netropy equation Eq.8.14 for a reversible process becomes

\[ (S_2 - S_1)_{\text{tot}} = 0 = m_{\text{hot}}(s_2 - s_1) + m_{\text{cold}}(s_3 - s_1) \]

Use specific heats to evaluate the changes in s from Eq.8.25 and division by m

\[ C_{p,\text{hot}} \ln(T_2 / T_1) - R \ln(P_2 / P_1) + C_{p,\text{cold}} \ln(T_3 / T_1) - R \ln(P_3 / P_1) = 0 \]

\[ P_3 = P_1 \frac{T_3}{T_1} \quad \text{and} \quad P_2 = P_1 \frac{T_2}{T_1} \]

Now everything is in terms of T and C\textsubscript{p} = C\textsubscript{v} + R, so

\[ C_{v,\text{hot}} \ln(T_2 / T_1) + C_{v,\text{cold}} \ln(T_3 / T_1) = 0 \]

same \( C_v: \quad T_3 = T_1 \frac{T_2}{T_1} = 667 \text{ K}, \quad P_3 = 333 \text{ kPa} \]

\[ Q_{\text{cold}} = -1 Q_2 = m C_v (T_3 - T_1) = -2480, \]

\[ W_{\text{HP}} = 1 Q_2 + Q_{\text{cold}} = 1 Q_2 - 1 Q_3 = 1244 \text{ kJ} \]
8.78 We wish to obtain a supply of cold helium gas by applying the following technique. Helium contained in a cylinder at ambient conditions, 100 kPa, 20°C, is compressed in a reversible isothermal process to 600 kPa, after which the gas is expanded back to 100 kPa in a reversible adiabatic process.

a. Show the process on a $T$-$s$ diagram.

b. Calculate the final temperature and the net work per kilogram of helium.

c. If a diatomic gas, such as nitrogen or oxygen, is used instead, would the final temperature be higher, lower, or the same?

Solution:

a. 

![Diagram](image)

b. The adiabatic reversible expansion gives constant $s$ from the entropy equation Eq. 8.14. With ideal gas and constant specific heat this gives relation in Eq. 8.32

$$T_3 = T_2 \left(\frac{P_3}{P_2}\right)^{\frac{k-1}{k}} = 293.15 \left(\frac{100}{600}\right)^{0.4} = 143.15 \text{ K}$$

The net work is summed up over the two processes. The isothermal process has work as Eq. 8.41

$$w_1 = -RT_1 \ln\left(\frac{P_2}{P_1}\right) = -2.0771 \times 293.15 \times \ln(600/100) = -1091.0 \text{ kJ/kg}$$

The adiabatic process has a work term from energy equation with no $q$

$$w_2 = C_v(T_2-T_3) = 3.116 (293.15 - 143.15) = +467.4 \text{ kJ/kg}$$

The net work is the sum

$$w_{\text{NET}} = -1091.0 + 467.4 = -623.6 \text{ kJ/kg}$$

c. Diatomic gas:

$$k < 1.67 \quad \text{(probably 1.40)}$$

From Eq. 8.32 $\Rightarrow T_3 > 143.2 \text{ K}$

$\Rightarrow \text{ Higher}$
A device brings 2 kg of ammonia from 150 kPa, -20°C to 400 kPa, 80°C in a polytropic process. Find the polytropic exponent, n, the work and the heat transfer. Find the total entropy generated assuming a source at 100°C.

Solution:

C.V. Ammonia of constant mass \( m_2 = m_1 = m \) out to source.

Energy Eq. 5.11: \( m(u_2 - u_1) = \dot{Q}_2 - \dot{W}_2 \)

Entropy Eq. 8.14, 8.18: \( m(s_2 - s_1) = \int dQ/T + \dot{S}_{2\text{ gen}} = \dot{Q}_2/T + \dot{S}_{2\text{ gen}} \)

Process: \( P_1 v_1^n = P_2 v_2^n \) \( \text{Eq. (8.36)} \)

State 1: Table B.2.2 \( v_1 = 0.79774 \quad s_1 = 5.7465 \quad u_1 = 1303.3 \)

State 2: Table B.2.2 \( v_2 = 0.4216 \quad s_2 = 5.9907 \quad u_2 = 1468.0 \)

\[
\ln \left( \frac{P_2}{P_1} \right) = \ln \left( \frac{v_2}{v_1} \right)^n = n \times \ln \left( \frac{v_2}{v_1} \right)
\]

\[
\ln \left( \frac{480}{150} \right) = n \times \ln \left( \frac{0.4216}{0.79774} \right) = 0.98083 = n \times 0.63773
\]

\( \Rightarrow n = 1.538 \)

The work term is integration of PdV as done in text leading to Eq. 8.38

\[
\dot{W}_2 = \frac{m}{1 - n} \left( P_2 v_2^n - P_1 v_1^n \right)
\]

\[
= \frac{2}{1 - 1.538} \times (400 \times 0.4216 - 150 \times 0.79774) = -182.08 \text{ kJ}
\]

Notice we did not use Pv = RT as we used the ammonia tables.

\[
\dot{Q}_2 = m(u_2 - u_1) + \dot{W}_2 = 2 \times (1468 - 1303.3) - 182.08 = 147.3 \text{ kJ}
\]

From Eq. 8.18

\[
\dot{S}_{2\text{ gen}} = m(s_2 - s_1) + \dot{Q}_2/T
\]

\[
= 2 \times (5.9907 - 5.7465) - 147.3 / 373.15
\]

\[= 0.0936 \text{ kJ/K} \]
8.94 A cylinder/piston contains air at ambient conditions, 100 kPa and 20°C with a volume of 0.3 m³. The air is compressed to 800 kPa in a reversible polytropic process with exponent, $n = 1.2$, after which it is expanded back to 100 kPa in a reversible adiabatic process.

a. Show the two processes in $P-v$ and $T-s$ diagrams.
b. Determine the final temperature and the net work.
c. What is the potential refrigeration capacity (in kilojoules) of the air at the final state?

Solution:

\[ P \\
\begin{array}{c}
2 \\
3 \\
1
\end{array} \]

\[ v \]

\[ m = \frac{P_1 V_1}{RT_1} = \frac{100 \times 0.3}{0.287 \times 293.2} = 0.3565 \text{ kg} \]

b) The process equation is expressed in Eq. 8.37

\[ T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = 293.2 \left( \frac{800}{100} \right)^{0.167} = 414.9 \text{ K} \]

The work is from Eq. 8.38

\[ W_2 = \int \frac{P dv}{1-n} = \frac{R(T_2-T_1)}{1-n} = \frac{0.287(414.9-293.2)}{1.120} = -174.6 \text{ kJ/kg} \]

Isentropic relation is from Eq. 8.32

\[ T_3 = T_1 \left( \frac{P_3}{P_2} \right)^{\frac{k-1}{k}} = 414.9 \left( \frac{100}{800} \right)^{0.286} = 228.9 \text{ K} \]

With zero heat transfer the energy equation gives the work

\[ 2W_3 = C_V(T_2 - T_3) = 0.717(414.9 - 228.9) = +133.3 \text{ kJ/kg} \]

\[ w_{NET} = 0.3565(-174.6 + 133.3) = -14.7 \text{ kJ} \]

c) Refrigeration: warm to $T_3$ at constant $P$ so

\[ Q_1 = m(u_1 - u_2) + W_1 = m(h_1 - h_3) \]

\[ Q_1 = mC_p(T_1 - T_3) = 0.3565 \times 1.004 (293.2 - 228.9) = 23.0 \text{ kJ} \]