Problem #1

Consider the spherical thrust bearing of Figure 1. The gap between the φ100-mm spherical member on the shaft and the housing is 0.25 mm and contains SAE 30 oil. The operating temperature is 30°C. The housing angle α is 90 degrees. The shaft rotates at 100 revolutions per second.

a) Find the shear stress that acts on the surface of the spherical member of the shaft as a function of position on the spherical surface.

b) Calculate the viscous torque.

Given: Dynamic viscosity of SAE 30 oil at 30°C: \( \mu = 2.0 \cdot 10^{-1} \frac{\text{kg}}{\text{m s}} \)

Figure 1: Spherical Thrust Bearing.
Given: Rotating bearing shown: 
narrow gap filled with viscous oil

\[ \mu \::= \cdot 2 \cdot 10^{-1} \text{kg/m/s} \]  
\[ R \::= 50 \text{mm} \]  
\[ \omega \::= 2 \cdot \pi \cdot 100 \text{rad/min} \]  
\[ h \::= 0.25 \text{mm} \]  
\[ \theta_{\text{max}} \::= 90 \text{deg} \]

Find: 
(a) shear stress on the surface of spherical member  
(b) calculate the viscous torque

Solution: Apply definitions

Basic equations:

Shear stress: \[ \tau = \mu \cdot \frac{du}{dy} \]

Elemental shear force: \[ dF = \tau \cdot dA \]

Area element: \[ dA = 2 \cdot \pi \cdot r \cdot ds \]

Arc segment: \[ ds = R \cdot d\theta \]

Elemental Torque: \[ dT = r \cdot dF \]

Assumption: 
(1) Newtonian fluid;  
(2) Narrow gap;  
(3) laminar motion

From the figure,

\[ r(\theta) := R \cdot \sin(\theta) \]  
\[ u(\theta) := \omega \cdot r(\theta) \]  
\[ \omega = 10.472 \text{Hz} \]

\[ \tau(\theta) := \frac{\mu \cdot \omega \cdot R \cdot \sin(\theta)}{h} \]

\[ dA = 2 \cdot \pi \cdot R \cdot \sin(\theta) \cdot R \cdot d\theta \]  
\[ dF = 2 \cdot \pi \cdot \tau \cdot R \cdot \sin(\theta) \cdot R \cdot d\theta \]
Thus the elemental Torque becomes:

\[ dT = R \cdot \sin(\theta) \cdot 2\pi \cdot \tau \cdot R \cdot \sin(\theta) \cdot R \cdot d\theta \]

\[ dT = 2\pi \cdot \mu \cdot \omega \cdot R^4 \cdot \frac{\sin(\theta)^3}{h} \cdot d\theta \]

\[ \theta := 0, 0.01.. \theta_{\text{max}} \]

The torque is:

\[ T := \int_0^{\theta_{\text{max}}} \frac{2\pi \cdot \mu \cdot \omega \cdot R^4 \cdot \sin(\theta)^3}{h} \cdot d\theta \]

\[ T = 0.219 \text{N-m} \]

or after analytical integration

\[ T := \frac{2\pi \cdot \mu \cdot \omega \cdot R^4}{3h} \cdot \left(1 - \cos(\theta_{\text{max}})\right)^2 \cdot \left(2 + \cos(\theta_{\text{max}})\right) \]

\[ T = 0.219 \text{N-m} \]
Problem #2

Consider the manometer shown in Figure 2, which is connected to a pipe carrying water. The manometer contains a liquid that has a specific gravity of 1.20. Find the relationship between h and H so that the pressure at A is equal to the pressure at B.
Problem 2

**Given:** Manometer connected as shown in Figure 2

\[ P_A = P_B \quad z_2 - z_1 = h \quad z_A - z_B = H \]

\[ SG_m := 1.2 \]

Manometer fluid density: \( \rho_m = SG \rho_w \)

Water density: \( \rho_w \)

**Find:** (a) find the relationship between \( h \) and \( H \) so that the pressure at A is equal to the pressure at B

**Solution:** Apply basic equation of hydrostatics.

\[ \frac{dP}{dz} = -\rho \cdot g \]

between points A and 1

\[ P_A - P_1 = -\rho_w \cdot g \cdot (z_A - z_1) \]

between points 1 and 2

\[ P_1 - P_2 = -\rho_m \cdot g \cdot (z_1 - z_2) = \rho_m \cdot g \cdot h \]

between points 2 and B

\[ P_2 - P_B = -\rho_w \cdot g \cdot (z_2 - z_B) \]

Add the above equations together

\[ P_A - P_B = -\rho_w \cdot g \cdot (z_A - z_1) + \rho_m \cdot g \cdot h - \rho_w \cdot g \cdot (z_2 - z_B) \]

So

\[ 0 = -\rho_w \cdot H - \rho_w \cdot h + \rho_m \cdot h \]

\[ h = \frac{H}{SG_m - 1} \]

\[ h = 5H \]