Given: Soda can will estimated dimensions $d = 66.0 \pm 0.5 \text{ mm}$, $h = 110 \pm 0.5 \text{ mm}$. Soda has SG $= 1.035$

Find: (a) volume of soda in the can (based on measured mass of full and empty can).
   (b) estimate average depth to which the can is filled and the uncertainty in the estimate.

Solution:

Measurements on a can of coke give

$h = 368.5 \pm 0.5 \text{ g}$, $m = 755 \pm 0.5 \text{ g}$

$u_m = \pm \left( \left( \frac{u_h}{h} \right)^2 + \left( \frac{u_m}{m} \right)^2 \right)^{1/2}$

$u_h = \pm \frac{0.5}{368.5} = \pm 0.00129 \text{ g}$, $u_m = \pm 0.5 \text{ g}$

$u_m = \pm \left( \left( \frac{368.5}{369} \right)^2 + \left( \frac{0.5}{369} \right)^2 \right)^{1/2} = 0.0019$ \text{ g}

Density is mass per unit volume and SG = $\rho / \rho_{\text{H}_2 \text{O}}$ so

$h = \frac{m}{\rho_{\text{H}_2 \text{O}} SG} = \frac{369 \text{ g}}{1.035 \times 10^3 \text{ kg/m}^3} = 350 \times 10^{-3} \text{ m}^3$

The reference value of $\rho_{\text{H}_2 \text{O}}$ is assumed to be precise, since SG is specified to three places beyond the decimal point, assume $u_{\rho_{\text{H}_2 \text{O}}} = \pm 0.001$. Then

$u_h = \pm \left( \left( \frac{1}{h} \right)^2 + \left( \frac{u_h}{\rho_{\text{H}_2 \text{O}} SG} \right)^2 \right)^{1/2} = \pm \left[ \left( \frac{1}{h} \right)^2 + \left( \frac{u_{\rho_{\text{H}_2 \text{O}}}}{\rho_{\text{H}_2 \text{O}} SG} \right)^2 \right]^{1/2}$

$u_h = \pm \left( \left( \frac{1}{350 \times 10^{-3}} \right)^2 + \left( \frac{0.001}{1.035 \times 10^3} \right)^2 \right)^{1/2} = 0.0021 \text{ or } 0.0019$ \text{ m}

$h = \frac{4}{\pi} \text{ m}$ or $h = \frac{4}{\pi} \frac{350 \times 10^{-3} \text{ m}^3}{0.036 \text{ m}^3} = \frac{4}{\pi} \times 102 \text{ mm} = 102 \text{ mm}$

Note: (1) printing on the can states the content as 355 ml. This suggests that the implied accuracy of the SG value may be overstated.
   (2) results suggest that over seven percent of the can height is filled with soda.