Given: Velocity field \( \mathbf{v} = Bx(1+At)c + cy^2 \), with \( A = 0.5 \text{s}^{-1} \), \( B = c = 1 \text{m} \), coordinates measured in meters.

Plot: The pathline of the particle that passed through the point \((1, 1, 0)\) at time \(t = 0\). Compare with the streamlines through the same point at the instants \(t = 0, 1, \) and \(2\) s.

Solution:

For a particle, \( u = \frac{dx}{dt} \) and \( v = \frac{dy}{dt} \)

Then

\[
\begin{align*}
Bx(1+At) &= \frac{dt}{dt} \\
\ln x &= B \left[ t + \frac{1}{2} At^2 \right]_0 \\
&= B \left[ t + \frac{1}{2} At^2 \right]_0 \\
v &= cy = \frac{dy}{dt}, \ 
\int c dt = \int \frac{dy}{cy} \ 
\int y = y_0 e^{ct} \end{align*}
\]

The pathline may be plotted by varying \( t \) as shown below. The streamline is found (at given \( t \)) from \( \frac{dy}{dx} \) streamline = \( u/v \).

Then \( \frac{dy}{dx} \) = \( \frac{cy}{Bx(1+At)} \) and \((1+At)\frac{dy}{dt} = \frac{c}{B} \frac{dt}{dt} \)

\( (1+At) \ln y = \frac{c}{B} \ln x + \ln c, \quad c, x \in \mathbb{R} \)

Streamline through point \((1, 1, 0)\) gives \( c_1 = 1 \). Then on substituting for \( A, B, \) and \( c \) we obtain

\[ x = y(1.05t) \]

At \( t = 0 \), \( x = y \)
\( t = 1 \text{s}, \ x = y \)
\( t = 2 \text{s}, \ x = 2y \)

![Graph showing streamlines and pathline](image)