Given: Pair of plane gates close a channel of width, \( W = 110 \text{ ft} \); each gate is hinged at channel wall. Gate edges are forced together at the channel center by water pressure. Water depth, \( D = 32 \text{ ft} \). Neglect the weight of the gate.

Find: (a) force exerted by water on gate A. 
(b) force components exerted by the gate on hinge A.

Solution:

Basic equations:
\[ dh = p \gamma ; \quad P = P_{\text{atm}} + pgh \]

Assumptions: 
1. static liquid  
2. gravity only body force  
3. \( + \) positive down from free surface  
4. \( + \) acts on both sides of gate

Then
\[ F_r = \left( p \Delta h \right) \Delta A = \frac{pgW}{2} \times 32.2 \text{ ft} \times 110 \text{ ft} \times 32 \text{ ft} \times \frac{1 \text{ slg}}{A \text{ slg}} \]
\[ F_r = 1.82 \times 10^6 \text{ lb} \]

Since the gate width, \( b = \frac{W}{2} \cos 15^\circ = 96.94 \text{ ft} \), is constant the line of action of \( F_r \) is located at \( b/2 \) from the hinge.

To find the reaction forces at the hinge, consider a free body of the gate. The correct forces at the hinge have only \( F_n \) and \( F_a \) components. The contact force \( F_n \), between the pair of gates must act perpendicular to the channel walls (from symmetry conditions).

\[ \sum F_x = 0 = F_r \cos 15^\circ - F_n \sin 15^\circ \]
\[ \therefore F_n = \frac{F_r}{\sin 15^\circ} = \frac{1.82 \times 10^6 \text{ lb}}{2 \sin 15^\circ} = 3.52 \times 10^6 \text{ lb} \]

\[ \sum F_y = F_r \sin 15^\circ - F_a = 0 \]
\[ \therefore F_a = F_r \sin 15^\circ = 1.82 \times 10^6 \text{ lb} \cos 15^\circ = 1.76 \times 10^6 \text{ lb} \]

The force on the hinge (from the gate) is \( F_h = (1.76 \times 10^6 + 3.04 \times 10^6) \text{ lb} \).