Given: The instantaneous leakage mass flow rate \( \dot{m} \) from a bicycle tire is proportional to the air density \( \rho \) in the tire and to the gauge pressure \( p \) in the tire.
Air in the tire is nearly isothermal (because the leakage rate is slow).
The initial air pressure is \( p_0 = 0.60 \text{ mPa (gage)} \) and the initial rate of pressure loss is \( 1 \text{ mPa/day} \).

Find: (a) Pressure in the tire after 30 days
(b) Accuracy of rule of thumb which says a tire loses pressure at the rate of "a pound 1 psl per day."

Plot: the pressure as a function of time over the 30 days; show rule of thumb results for comparison.

Solution:

Apply conservation of mass to tire as the \( CV/\dot{m} \)
Basic equation:

\[
\dot{m} = \frac{\partial c}{\partial t} + \int_0^r \frac{\partial c_p}{\partial p} dp
\]

Assumptions:
(a) uniform properties in tire
(b) our inside CV behaves as ideal gas
(c) \( T = \text{constant} \)
(d) \( n = c (p - P_{\text{atm}}) p \)

Then we can write

\[
0 = \frac{\partial c}{\partial t} + \int_0^r \frac{\partial c_p}{\partial p} dp
\]

But \( p = nRT \) and \( \frac{\partial c_p}{\partial p} = \frac{c_p}{RT} \), so

\[
0 = \frac{\partial c}{\partial t} + \frac{c_p}{RT} (p - P_{\text{atm}})
\]

At \( t = 0 \), \( p = p_0 \) and \( \frac{\partial c}{\partial t} = \frac{\partial c}{\partial t} \). Plus

\[
0 = \frac{\partial c}{\partial t} + \frac{c_p}{RT} (p_0 - P_{\text{atm}}) \quad \text{and} \quad c = -\frac{\dot{m}_0}{p_0(P_{\text{atm}} - P_0)} \frac{\partial c}{\partial t}
\]

Substituting into Eq. \( \text{1} \) we obtain

\[
0 = \frac{\partial c}{\partial t} + \frac{p - (p - P_{\text{atm}})}{p_0(P_{\text{atm}} - P_0)} \frac{\partial c}{\partial t}
\]

Separating variables and integrating

\[
\int_0^{p_{\text{atm}}} \frac{\partial c}{c(p - P_{\text{atm}})} = \int_0^{t} \frac{\partial c}{c(p - P_{\text{atm}})}
\]

\[
\frac{1}{P_0} \int_0^{P_{\text{atm}}} \frac{p(p - P_{\text{atm}})}{c(p - P_{\text{atm}})} = \frac{\partial c}{\partial t} \quad \text{and} \quad c = -\frac{\dot{m}_0}{p_0(P_{\text{atm}} - P_0)} \frac{\partial c}{\partial t}
\]

\[
\dot{m} = \frac{1}{P_0} \int_0^{P_{\text{atm}}} \frac{p(p - P_{\text{atm}})}{c(p - P_{\text{atm}})} = \frac{\partial c}{\partial t} \quad \text{and} \quad c = -\frac{\dot{m}_0}{p_0(P_{\text{atm}} - P_0)} \frac{\partial c}{\partial t}
\]

\[
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\]
Taking antilog of

\[ 1 - \frac{P_{cm}}{P_0} = \left(1 - \frac{P_{cm}}{P_0}\right) e^{\frac{dP/dt}{P_0 (P_0 - P_{cm})}} = \left(1 - \frac{P_{cm}}{P_0}\right) e^{kt} \]

where

\[ k = \frac{dP/dt}{P_0 (P_0 - P_{cm})} = \frac{1}{\rho \text{air} \times 6.895 \text{kPa}} \times \frac{1}{101.325 \text{kPa} \times (101.325 - P_{cm})} \]

\[ t = 0.00116 \text{ day}^{-1} \]

and

\[ \phi = \frac{P_{cm}}{1 - \left(\frac{P_0 - P_{cm}}{P_0}\right) e^{kt}} \]

Evaluating at \( t = 30 \text{ days} \)

\[ \phi = \frac{101.325 \text{kPa}}{1 - \frac{101.325 \text{kPa}}{101.325} e^{-30(0.00116 \text{ day}^{-1})}} = 544 \text{ kPa} \quad P_{cm} \text{ at 30 days} \]

Rule of Thumb gives \( \phi = P_0 - 6.895 \text{kPa}/\text{day} \)

At \( t = 30 \text{ days} \)

\[ \phi = 600 \text{kPa} - 201 \text{kPa} = 403 \text{kPa} \quad P_{cm} \]

The rule of thumb predicts a larger pressure loss.

Results for both models are presented below.

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![Tire Pressure vs. Time](image-url)

**Tire Pressure vs. Time**

- **Exact Model**
- **Rule-of-Thumb**

![Graph](image-url)