Problem-9.17

The square test section of a small laboratory wind tunnel has sides of width \( W=305 \text{ mm} \). At one measurement location, the turbulent boundary layers on the tunnel walls are \( \delta_1=9.5 \text{ mm} \) thick. The velocity profile is approximated well by the "1/7-power" expression. At this location the freestream air speed is \( U_1=18.3 \text{ m/s} \), and the static pressure is \( p_1=-22.9 \text{ mm H}_2\text{O}(gage) \). At a second measurement location downstream, the boundary layer thickness is \( \delta_2=12.7 \text{ mm} \). Evaluate the air speed in the freestream at the second location. Calculate the difference in static pressure from section (1) to section (2).

Given:
Air flow in laboratory wind tunnels.

\[
p_1 := -22.9 \text{ mm H}_2\text{O}
\]

\[
U_1 := 18.3 \text{ m/s}
\]

\[
\delta_1 := 9.5 \text{ mm}
\]

\[
\delta_2 := 12.7 \text{ mm}
\]

\[
W := 305 \text{ mm}
\]

Density of Air:
\[
\rho := 1.23 \frac{\text{kg}}{m^3}
\]

Density of Water:
\[
\rho_w := 997 \frac{\text{kg}}{m^3}
\]

Find:
(a) Freestream speed at (2)
(b) Pressure difference, \( \Delta p = p_1-p_2 \)

Solution:
Apply displacement thickness, continuity and Bernoulli equations

Basic equations:
\[
\delta^*/\delta = \int_0^1 \left( 1 - \frac{u}{U} \right) \frac{d}{dy} \left( \frac{y}{\delta} \right) dy
\]

\[
\frac{p_1}{\rho} + \frac{U_1^2}{2} + g_y \frac{\delta^*}{\delta} = \frac{p_2}{\rho} + \frac{U_2^2}{2} + g_y \frac{\delta}{\delta}
\]

Assumptions:

(1) steady flow
(2) Incompressible flow
(3) No friction (outside BL)
(4) Along a streamline
(5) Uniform flow (outside BL)
(6) Same BL on four surfaces
(7) Neglect corner effects
(8) Neglect \( \Delta z \)

Then
\[ u_1 = \frac{u}{U}, \quad \eta = \frac{y}{\delta}, \quad u_1(\eta) := \frac{1}{7} \]

\[ \delta^*/\delta = D_1 \]

\[ D_1 := \int_0^1 \left( 1 - \eta \right) \frac{1}{\eta} \, d\eta \]

\[ D_1 = 0.125 \]

\[ \delta_{1\text{star}} := \delta_1 \cdot D_1 \quad \delta_{1\text{star}} = 1.191 \text{ mm} \]

\[ \delta_{2\text{star}} := \delta_2 \cdot D_1 \quad \delta_{2\text{star}} = 1.592 \text{ mm} \]

From continuity,

\[ U_1 A_1 = U_1(W-2\delta_1^*)^2 = U_2 A_2 = U_2(W-2\delta_2^*)^2 \]

\[ U_2 := U_1 \left( \frac{W - 2 \cdot \delta_{1\text{star}}}{W - 2 \cdot \delta_{2\text{star}}} \right)^2 \]

\[ U_2 = 18.397 \frac{\text{m}}{\text{s}} \]

From the Bernoulli Equation

\[ \frac{p_1}{\rho} + \frac{U_1^2}{2} = \frac{p_2}{\rho} + \frac{U_2^2}{2} \]

\[ \Delta p = p_1 - p_2 \]

\[ \Delta p := \frac{\rho}{2} \left( U_2^2 - U_1^2 \right) \]

\[ \Delta p = 2.199 \text{ Pa} \]

Since, \[ \Delta p = \rho \cdot g \Delta h \]

\[ \Delta h := \frac{\Delta p}{\rho \cdot g} \]

\[ \Delta h = 0.225 \text{ mm} \text{ } \text{H}_2\text{O} \]