Problem 10.10
Consider the centrifugal pump impeller dimension in Example Problem 10.1. Construct the velocity diagram for shockless flow at the impeller inlet if \( b = \) constant. Calculate the effective flow angle with respect to the radial impeller blades for the case of no inlet swirl. Investigate the effects on flow angle of (a) variations in impeller width and (b) inlet swirl velocities.

Given:
Impeller dimension of Example Problem 10.1
\[
Q := 150 \frac{\text{gal}}{\text{min}} \quad D_1 := 1.25 \text{ in} \\
b_1 := 0.383 \text{ in} \quad N := 3450 \text{ rpm}
\]

Find:
Construct velocity diagram for shockless flow at the impeller inlet. Investigate effects on inlet flow angle of
(a) variations in impeller width
(b) variations in inlet swirl velocity

Solution:
\[
\rho := \frac{D_1}{2} \quad \omega := \frac{(2 \pi N)}{60} \\
\omega = 361.283 \frac{\text{rad}}{\text{sec}}
\]

From continuity,
\[
V_{n1} := \frac{Q}{2 \pi r_1 b_1} \quad U_1 := \omega r_1
\]
\[
V_{n1} = 31.997 \frac{\text{ft}}{\text{sec}} \quad U_1 = 18.817 \frac{\text{ft}}{\text{sec}}
\]
\[
\beta_1 := \text{atan} \left( \frac{V_{n1}}{U_1} \right) \\
\beta_1 = 59.541 \text{ deg}
\]

Thus for radial vanes,
\[
\theta_{\text{eff}} = \left( \frac{\pi}{2} \right) - \beta_1
\]
\[
\theta_{\text{eff}} := 90.\text{deg} - \beta_1
\]
\[
\theta_{\text{eff}} = 30.459 \text{ deg}
\]

Effective flow angles for radial blades, \( \theta_{\text{eff}} = 30.459 \text{ deg} \)
(a) Vary impeller width, \( b \) with no inlet swirl

\[
\begin{align*}
Q &= 0.334 \text{ ft}^3/\text{s} \\
r_1 &= 0.0521 \text{ ft} \\
U_1 &= 18.8 \text{ ft/s}
\end{align*}
\]

<table>
<thead>
<tr>
<th>( b_1 ) (ft)</th>
<th>( V_1 ) (ft/s)</th>
<th>( \beta_1 ) (deg)</th>
<th>( \theta_{\text{eff}} ) (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>102.0302</td>
<td>79.56218</td>
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<td>0.032</td>
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</tbody>
</table>

(b) Vary inlet swirl (\( V_{t1} \)) with \( b = \) constant = 0.0319 ft

\[
\beta_1 = \tan^{-1} \left( \frac{V_{n1}}{U_1 - V_{t1}} \right)
\]

<table>
<thead>
<tr>
<th>( V_{t1} ) (ft/s)</th>
<th>( \beta_1 ) (deg)</th>
<th>( \theta_{\text{eff}} ) (deg)</th>
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</thead>
<tbody>
<tr>
<td>0</td>
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<td>15</td>
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<td>6.77216</td>
</tr>
<tr>
<td>18.8</td>
<td>90</td>
<td>0</td>
</tr>
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</table>
with respect to the radial impeller blades for the case of no inlet swirl. Investigate the effects on flow