**Problem 12.19:**

**Given:** Isentropic flow of air from a large tank through a converging nozzle discharges to the atmosphere. The state in the plenum feeding the nozzle is designated as (0), the state at the entry to the converging nozzle is designated as (1), the state at the exit of the nozzle where the area is minimum is designated as (2), and the state at the discharge plenum (atmosphere) is designated as (b).

**Stagnation conditions:**

\[ P_0 := 171000 \text{ Pa} \quad T_0 := (273.15 + 27) \text{ K} \]

\[ M_1 := 0.2 \]

\[ A_2 := 0.015 \text{ m}^2 \quad R := 287 \text{ N} \text{ m} \text{ kg}^{-1} \text{ K}^{-1} \]

\[ P_{\text{atm}} := 101000 \text{ Pa} \quad P_b := P_{\text{atm}} \]

\[ k := 1.4 \]

**Find:** Magnitude and direction of the force required to keep the nozzle in place.

**Solution:**

Conservation of x-momentum for CV volume enclosing the nozzle

\[ P_1 g A_1 - P_2 g A_2 - R_x = m_{\text{rate}} (V_2 - V_1) \]

Conservation of mass

\[ m_{\text{rate}} = \rho \cdot V \cdot A = \text{constant} \]

Perfect gas

\[ P = \rho \cdot R \cdot T \]

Isentropic Flow Relations

\[ \frac{T_0}{T} = \left( 1 + \frac{k - 1}{2} M^2 \right)^\frac{k}{k-1} \quad \frac{P_0}{P} = \left( 1 + \frac{k - 1}{2} M^2 \right)^\frac{k}{k-1} \]

**Assumptions:**

1. steady flow
2. uniform flow at each section
3. isentropic flow
4. perfect gas
Therefore the flow is not choked

\[
M_2 := \left[ \frac{2}{k-1} \left( \frac{P_0}{P_b} \right)^{\frac{k-1}{k}} - 1 \right]^{\frac{1}{2}}
\]

\[
P_1 := \frac{P_0}{\left( 1 + \frac{k-1}{2} \cdot M_1^2 \right)^{k-1}}
\]

\[
P_2 := P_b
\]

\[
T_2 := \frac{T_0}{\left( 1 + \frac{k-1}{2} \cdot M_2^2 \right)}
\]

\[
a_2 := \sqrt{k R_a T_2}
\]

\[
V_2 := M_2 a_2
\]

\[
\rho_2 := \frac{P_2}{R_a T_2}
\]

\[
m_{\text{rate}} := \rho_2 V_2 A_2
\]

\[
T_1 := \frac{T_0}{\left( 1 + \frac{k-1}{2} \cdot M_1^2 \right)}
\]

\[
a_1 := \sqrt{k R_a T_1}
\]

\[
V_1 := M_1 a_1
\]

\[
P_1 = 1.663 \times 10^5 \text{ Pa}
\]

\[
V_1 = 69.179 \text{ m/s}
\]

\[
M_2 = 0.901
\]
\[ \rho_1 := \frac{P_1}{R_a T_1} \]

\[ \rho_1 = 1.946 \text{ kg/m}^3 \]

\[ A_1 := \frac{m_{\text{rate}}}{\rho_1 V_1} \]

\[ A_1 = 0.044 \text{ m}^2 \]

\[ R_x := (P_1 - P_{\text{atm}}) A_1 - m_{\text{rate}} (V_2 - V_1) \]

\[ R_x = 1.566 \times 10^3 \text{ N} \]