6.a. The single degree-of-freedom dynamic absorber

In 1911 H. Frahm has invented a device for stabilization of rocking oscillations of ships. This device is now called a *dynamic absorber*. It is one of the greatest inventions in vibration control, frequently used in applications. As most great inventions, it is based on a simple idea, described below.

Consider the single degree-of-freedom system, excited by harmonic force \( f(t) = F_0 \sin \omega t \) as shown below.

\[
f(t) = F_0 \sin \omega t
\]

The Primary System

This is our *primary* system. We know that the primary mass vibrates with two frequencies, the natural frequency of vibration \( \omega_n = \sqrt{k_p/m_p} \) and the forced frequency of vibration \( \omega \). Our objective is to eliminate the component of the forced vibration of the primary mass \( m_p \). This is achieved by attaching a secondary single degree-of-freedom system, with mass \( m_s \) and spring \( k_s \), to the primary system as shown below.

\[
f(t) = F_0 \sin \omega t
\]

The Controlled System

The problem is to choose the right parameters, \( m_s \) and \( k_s \), of the secondary system. The equations of motion for the controlled system are

\[
\begin{bmatrix}
    m_p & \dot{x}_p \\
    m_s & \dot{x}_s
\end{bmatrix}
+ \begin{bmatrix}
    k_p + k_s & -k_s \\
    -k_s & k_s
\end{bmatrix}
\begin{bmatrix}
    x_p \\
    x_s
\end{bmatrix}
= \begin{bmatrix}
    F_0 \\
    0
\end{bmatrix}\sin \omega t,
\] (6.1)

or in component form
The forced harmonic vibrations of the undamped system have the form
\[
\begin{align*}
    \mathbf{x}_p(t) &= X_p \sin \omega t \\
    \mathbf{x}_s(t) &= X_s \sin \omega t
\end{align*}
\] (6.3)
where \( X_p \) and \( X_s \) are some constants. Substitution (6.3) in (6.2) gives
\[
\begin{align*}
    -\omega^2 m_p X_p \sin \omega t + (k_p + k_s) X_p \sin \omega t - k_s X_s \sin \omega t &= F_0 \sin \omega t \\
    -\omega^2 m_s X_s \sin \omega t - k_s X_p \sin \omega t + k_s X_s \sin \omega t &= 0
\end{align*}
\] (6.4)
or, upon eliminating \( \sin \omega t \) throughout
\[
\begin{align*}
    -\omega^2 m_p X_p + (k_p + k_s) X_p - k_s X_s &= F_0 \\
    -\omega^2 m_s X_s - k_s X_p + k_s X_s &= 0
\end{align*}
\] (6.5)
Assuming that there are \( m_s \) and \( k_s \) which cause \( X_p \) to vanish, we substitute \( X_p=0 \) in (6.5) and obtain
\[
\begin{align*}
    -k_s X_s &= F_0 \\
    -\omega^2 m_s X_s + k_s X_s &= 0
\end{align*}
\] (6.6)
From the second equation of (6.6) we find that we must have
\[
\frac{k_s}{m_s} = \omega^2
\] (6.7)
and from the first equation of (6.6) we find that in this case the amplitude of vibration of the secondary mass \( m_s \) is
\[
X_s = -\frac{F_0}{k_s}
\] (6.8)
Since (6.6) has a solution we conclude that the assumption that there are \( m_s \) and \( k_s \) which cause \( X_p \) to vanish, is justified.
In summary, if we choose the mass and stiffness of the secondary system such that \( k_s/m_s = \omega^2 \), the component of the forced harmonic vibration of the primary mass \( m_p \) will vanish. In this case the amplitude of vibration of the secondary mass is \( X_s = -F_0/k_s \).

The physical interpretation of this result is that when the mass and stiffness of the secondary system satisfy the relation (6.7) the secondary system vibrates out of phase with the external force \( f(t) \). In this case the spring \( k_s \) applies force on \( m_p \) which exactly contradicts the force \( f(t) \), resulting the forced motion of \( m_p \) to vanish.

6.b. The multi degree-of-freedom dynamic absorber